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Competition and Regulation in the Railway industry

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By
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To my family
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Abstract

The railway industry has foregone important changes in the last years, due to the European reform aimed at its liberalization. A number of studies analyze the qualitative differences across countries, because each Member State has some degree of discretion as the European directives provide only for the main principles (Seabright, 2003; Newbery, 1999; Gomez-Ibanez, 2003; Gomez-Ibanez e De Rus, 2007). But as for the effect of the reform, the analysis are still limited (Friebel, 2008; Quinet, 2006; Schmutzler and Lalive, 2008) and also from a theoretical point of view, only few papers consider the railway industry (De Villemeur et al. 2003).

Therefore, the aim of the thesis is to investigate some interesting and important aspects of the railway sector. In particular, I first focus on the role of private or public ownership to incentive investments in rolling stock and second on the issue of public service obligations.

The thesis is organized as follows: the introduction illustrates main provisions of the European reform and highlights the characteristics of the sector.

The first chapter reviews the literature on public-private partnerships with reference to the issue of bundling or unbundling of different phases of a project. The interest for this kind of arrangement is related to the question of how to solve the problem of scarcity of rolling stock that appears to affect the competitive selection of the service provider. In the second chapter I present a model to study what the effect of the ownership of the rolling stock is on the incentive of the provider of the service to invest also in the case of asymmetric information over the cost. In this latter case a trade off emerges between the effect of the investment and the rent from asymmetric information.

In the second part of the thesis I analyze the issue of public service obligation. In chapter three I present a review of the literature about the universal service, a concept close to public service obligation but formally applied only in the telecommunications and in the postal sector. In the forth chapter, I consider the effect of PSO on total welfare according to different market structures and in particular when a PSO fund is set up. I analyze welfare effect both under complete information and under asymmetric information over demand. It appears that the set up of a PSO fund grants welfare gains also under asymmetric information.
Introduction

At the end of the 1980s and in the 1990s some member States of the European Union started to restructure the railway sector and to reform the regulatory framework in order to open up rail markets.

In order to understand the rationale for European reform, a brief overview of the characteristics of the sector is useful.

The demand for rail transport is usually defined as a derived demand, as it derives from the more general need of mobility. The utility of the consumers depends on both the preferences for the characteristics of the service, such as speed, comfort and reliability and also on the travel purpose. In economic analysis, consumers are usually distinguished into time sensitive or price sensitive, roughly corresponding to business traveller and leisure traveller.

Empirical studies on demand elasticity (see for example Winston, 1985; Oum et al., 1990) show very low values, at least in the short-run and this is true both when considering only rail services and competing mode of transport. This result suggests that once the consumer has chosen a type of service, he is not willing to change it in the short run.

However, in the long run demand elasticities are higher. An evidence of this result is the decrease in the share of railway in the transport sector, but it is difficult to understand how much of this change is due to modification of relative prices or to travel times.

The supply side is characterized by a high product differentiation and substitutability, as each route can be defined as a single product, but there also exist alternative as (in most of the cases) the same origin-destination can be connected using another path.

As for costs, variable costs are mainly fuel and maintenance expenditure, which depend on the speed and on the number of wagons, a proxy for the number of passengers. Fixed costs are related for example to crew wages, depreciation of the rolling stock and infrastructure costs charged to the train operator.

The literature distinguishes between economies of densities and economies of size (Seabright et al., 2003).
The first refer to the change in costs due to the increase in traffic, given the network. The latter represents the change in costs given the density, but varying the network.

Empirical analysis highlights the existence of economies of density, as if traffic increases on a certain network, the passenger – kilometre cost decreases, while the results for economies of size are not clear cut. For example, Cantos (2001) find that returns to density for the main European operators vary from 1.42 to 2.04, while returns to scale vary between 0.45 to 1.4.

Another important feature of railways are externalities, such as environmental and congestion externalities. If it is quite clear what we are referring to with environmental externalities, it is probably less obvious the concept of congestion.

Congestion occurs when the delay of a train induces delays for the other trains and it could be due to accidents or incidents on a line and it is strictly related to the rail capacity, whose use depends on the allocation among trains with different speed.

We now turn to the provisions of European directives.

At the European Community level, the White Paper in 1996 on rail transport laid down the strategic principles aimed at revitalising the railway sector in order to increase its competitiveness and attractiveness with customers. The Community Transport White Paper of 2001 defined a political target of maintaining the 1998 rail modal share by the year 2010. In order to reach this target the Community rail policy aims at ensuring non-discriminatory market access and transparent market structures, providing incentives for an efficient infrastructure use, contributing to a sustainable financial restructuring of railway undertakings and infrastructure managers, triggering a positive rail market development.

European Union countries agreed on the opening and integration of the formerly closed monopoly railway markets. In freight transport, an open access approach was chosen in order to obtain competition “on the tracks”. In passenger transport, the approach proposed by the European Commission consisted in open access for international services and in regulated competition, for instance, for urban services through the tendering of franchises or public service contracts (competition for the tracks).
Moreover, separation between infrastructure management and transport service provision is required: as a minimum, this separation must be done for the essential functions so as to ensure non-discriminatory network access such as capacity allocation and setting of track access charges.

In the European Union, the first step made to achieve fully open and integrated rail markets was the rail interoperability and rail infrastructure package directives of 2001.

The directive on interoperability of conventional rail systems (Directive 2001/16/EC) describes, similarly to the high-speed rail directive (Directive 96/48/EC), a process of technical harmonisation of the railway based on Technical Specifications for Interoperability.

The three directives of the infrastructure package (the first railway package), defined the access rights to use rail infrastructure for international freight services, the various conditions railway firms must fulfil to be able to benefit from the access rights, the independence of functions essential for ensuring non-discriminatory access and the possibilities of appeal that the market actors should have.

The second legislative railway package was adopted in 2004; it provided for full open access for all kinds of rail freight services, a common approach to European rail safety, extending the scope of the interoperability directives and the setting up of a European Railway Agency (ERA) with the aim of driving forward the technical implementation of the EU safety and interoperability approach.

In 2007, the European Commission adopted the third railway package, for the opening up of international passenger services. This package wants to improve the rights of passengers using international services, establishes a certification system for locomotive drivers and steps up the quality of freight services.

Each EU member can choose how to implement the regulatory framework, but they certainly have to respect basic principles as transparency and independence of essential infrastructure management functions for non-discriminatory access (e.g. capacity allocation, setting of track access charges).

A lot of studies compare and illustrate different kind of organization chosen by countries (Seabright, 2003; Newbery, 1999; Gomez-Ibanez, 2003; Gomez-Ibanez e De Rus, 2007).
As for the effect of the reform, the analyses are still limited. From a qualitative point of view, the liberalization index elaborated by IBM (2002, 2004, and 2007) gives a measure of how much markets have been liberalized. The liberalization index takes into account the level of access both in terms of what the law provides for and what the law in action is. According to this index all countries improved their liberalization level, even if there are strong differences among them.

The group of advanced countries is composed of Great Britain, Germany, Sweden and the Netherlands.

There are only few empirical studies about the effect of the reform. Friebel et al. (2008) use World Bank data from 1980 to 2003 to estimate the effect of open access, independent regulation and vertical separation on railway operators’ performance. They find a positive effect on technical efficiency around 0.5% per year and they also highlight the importance of sequential adoption of the reforms. The intuition for this latter result is quite clear as the sequential implementation gives the opportunity to better adapt the organization to the need of the sector. Unfortunately this data cannot measure the opening of the markets concretely, but only according to provisions of law.

Friebel et al. also study the evolution of the relative efficiency of European countries along time and it appears that The Netherlands are the most advanced countries. Together with The Netherlands, France, Denmark and Spain, Italy is among the countries that maintain a high position throughout the period.

Quinet’s analysis (2006) also controls for the effect of the high speed trains but the results are not far from Friebel et al..

The aim of this thesis is to analyze some aspects of the railway industry using the tools of the theory of incentives, because the relation between the regulator or a transport authority and the railway operator involves a delegation problem. In particular, we examine the issue of how to incentive investment in rolling stocks and how to satisfy public service obligations. Both this issues are subject to a problem of asymmetric information. In the first case the railway operator has private information over cost and in the second over demand.

The thesis is organized as follows: the first chapter presents a review of the literature about public-private partnership with particular reference to the case for separation or bundling of different phases of a project. In
the second chapter, we consider the incentive to invest in the railway industry if the rolling stocks are owned by the operator or by the public authority. The study is presented both in the case of complete information and with a problem of adverse selection concerning the operator selected to provide the service. It appears that it is not possible to define ex ante if public or private ownership induces more investment as it depends on the trade off between the cost of public funds and the distortion induced by asymmetric information.

The last two chapters are dedicated to the issue of public service obligations (PSO).

We first review the literature about universal service, mainly referred to the telecommunications and the postal sector and then present a model to illustrate the effect on welfare of different market structures if one of the firms is constrained by public service obligation.

One of the proposed solutions to the financing issue of the obligations is the set up of a public service obligations fund, where all the operators contribute with a share of their profits. Even under asymmetric information, the PSO fund leads to higher welfare.
Chapter 1
Public-Private Partnerships

1. Introduction

A public-private partnership (PPP) is a contractual agreement between the public and the private sectors, whereby the private operator commits to provide public services that have traditionally been supplied or financed by public institutions.

The rationale for using PPPs is usually claimed to be a series of advantages as, among others, the more efficient allocation of risk, faster implementation and the improvement in the quality of services. The adoption of PPP is also linked to the opportunity to exploit the specific abilities and efficiency of private sector, but controlling for the quality level of the service.

The adoption of PPPs is increasing. In the European Union (Riess, 2005) PPPs for roads, bridges, and tunnels account for about 83% of all PPP total value (31.5 billion for the period 1995-2003), with rail transport and airports making up 5% and 7%, respectively. Data from U.K. show that in railway sector PPPs account for about 51% of the total value of contracts and more than one third (37%) is represented by the three London Underground projects.

Even if from a quantitative point of view, public-private partnerships do not represent a substantial part of total investments, they certainly are very interesting from a qualitative point of view.

In what follows, we first propose an overview of the different models of partnerships. Then focus on the main insights of the literature about the bundling or unbundling of various phases of the project and finally look at how theory fits with empirical evidence.

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1 DG Internal Policies of the Union, 2006
2 The Channel Tunnel Rail Link account for about 12%, but it is no longer listed as PPP by the U.K. government
2. Models of PPP

There exists a wide range of agreements belonging to PPPs. According to the Green Paper (2004) they can be divided into institutional and contractual PPPs.

The first type of agreement entails the creation of a new entity which is jointly held by the public and the private operators.

If there is only an agreement between the private and the public sector to provide a service in exchange for some form of compensation, then it is a contractual PPP.

One of the main features is how the risk is shared between private and public sector and in general among parties.

The assessment of what kind of risk is involved at which stage is central in the evaluation of the project.

In the building stage, concerns may emerge about the delay of completion of the project and therefore of the beginning of cash flows or about cost overruns.

At the operating phase difficulties may emerge because of traffic or revenues not in line with previous forecasts or because of international contingencies that affect interest rate or exchange rate.

To sum up, with the expression risk we refer not only to the demand or the financial risk, but also to the construction risk, related to the design and construction phase, to the performance or availability risk related to the delivery or availability of the assets as defined by the contract and the residual value risk related to the future market price of the assets.

Besides these business risks, there are risks associated with the interactions with the public sector as for example changes in regulatory or legal environment.

In the rest of this section, we describe the phases that usually compose a PPP and which are the feasible arrangements.

2.1. The steps of a Public-Private Partnership

Different models of PPPs can be set up. Even if each of them has its specificities, we can identify the main steps and which issues can arise.
The entire process can be divided into:

- tendering,
- building and financing,
- operational phase,
- renegotiation.

In the tendering phase, the public partner has to define exactly which service or assets have to be delivered. It has to give a description of both the outcome and of the quality.

If the quality is measurable then it should establish standards or benchmarks.

But in some cases it is not possible to measure it, so a valid alternative is the so-called competitive dialogue. With this procedure, introduced with Directive 2004/18/EC, all bidders have to submit their own solution to deliver the assets or the service. Main concerns with competitive dialogue are related to the incentive for the bidder to reveal detailed information if he thinks that the idea will be implemented by one of the other bidder. This issue explains why competitive dialogue is often a very long and complex procedure.

In the building phase the main problem is related to possible wrong estimation of costs, while the financing phase is more complex and subject to numerous risks as we will see in more details later.

In the operational phase, the private party usually plays the main role, but the public sector keeps on monitoring the activity.

Finally, consider the renegotiation phase. It could be a scheduled or an unforeseen renegotiation. Scheduled renegotiations take place on the basis of the original arrangements between parties; unforeseen renegotiation depends on contingencies, as for example when there are problems of overestimation of demand.

### 2.2. Alternative forms of Public-Private Partnership

To have an idea of the large variety of agreements that can occur, consider that the European Commission lists the main categories as service contract, operation and management contracts, leasing agreements, Build-Operate-Transfer and Design-Build-Finance-Operate (DBFO) (DG Internal Policies, 2006).
• Service contract

The private partner of a service contract is usually required to procure, operate and maintain assets for a short period of time. It is the case for example of the provision and maintenance of technical activities or of toll collection service.

The public sector bears the financial and the residual value risk, so that its main advantage resides in the possibility to benefit from specific skills of the private operator.

• Operation and management contract

If the responsibility of assets operation and management is passed to the public sector, then the agreement is of the second type: operation and management contract.

Public sector still retains investment and financial risk. The advantage of this agreement is that the private party is paid according to a fixed fee or to an incentive basis linked to specific performance targets. Therefore, the private partner is strongly interested in improving service quality to reduce both overall costs and the demand risk.

• Leasing agreement

The third kind of arrangement is the leasing agreement: the private agent benefits of the income streams generated by publicly owned assets in exchange for a fixed lease payment and the obligation to operate and maintain the assets.

In this case, the private provider bears the commercial and the demand risk and this explain why it has an incentive to achieve operational efficiency. It is also in charge of the risks related to network expansion, capital improvements and financing.

• Build-Operate-Transfer

With the Build-Operate-Transfer (BOT) (or Turnkey procurement) the public party is still in charge of the financial risk and the private partner has to build and operate the assets and then transfer it to the public authority. The bundle of different stages induces the operator to carefully consider the operating cost and the design and operation phase. On the other hand, as the public partner gives up control rights until the transfer at the end of the contract, it is essential that output and quality specifications are well defined.
Design-Build Finance-Operate

Finally in Design-Build Finance-Operate (DBFO), the private partner designs the project on the basis of the requirements set by the public entity, ensures and finances the completion of the assets and operates the facility.

As soon as the contract expires, the service or assets can be transferred to the public sector as initially established or the agreement can be renegotiated. This kind of PPP has the advantages of BOT and also provides for the source of capital.

Within the DBFO agreement it is possible to distinguish concession and private divestiture.

With a DBFO concession the private investor obtains the right to collect the revenues over a specified period of time. Ownership of the assets remains with the public sector.

To the contrary, in private divestiture the assets are partially or entirely sold to the private sector. The government has a regulatory role aimed at protecting consumers from monopolistic prices and output restrictions.

As the above classification highlights, there are different models of PPPs and each of them can be adapted to the need of the sector. Moreover, the agreement is not only sector specific but also activity specific: some models are better suited than other in reaching certain objectives and managing the risk characterizing the project.

A central role in the PPPs in the transport sector is played by project finance, “typically used in those sectors that require large capital expenditures, that have long-lived assets, and that require long periods to amortize investment costs and generate required rates of return for both creditors and equity holders.” (Estache et al., 2007)

In most of the cases, the source of finance is not unique and can take different forms, such as equity, commercial lending, bond finance, etc.

The adoption of project finance is obviously linked to the existence of some benefits that affect the partners according to their role in the project. For example, firms can limit their financial risk in a project to the amount of their equity investment. Moreover, as the PPP requires a very careful assessment of the project, the risk among parties is shared more appropriately, than in other kind of arrangements.
On the other hand the complexity of the project and the high number of parties involved definitely increases transaction costs, both before (agreement of parties, consultancy, etc.) and after the building stage (monitoring).

Estache et al. (2007) highlight that the role of the public sector is not limited to the contractual partnership, as it also acts as provider of guarantees and to monitor contractual commitments.

As for guarantees, a wide range of mechanisms can be implemented: equity and debt guarantees gives respectively to the private operator and to the lenders the option to be bought out by the government at a price that guarantees a minimum return on equity.

Specific clauses can also be set in order to reduce demand risk, especially in toll roads projects: with traffic or a revenue guarantee, the government compensates the private partner if traffic or revenues fall below a defined threshold. An alternative is a variable length of the contract.

Another role of the public sector is to monitor PPP. It could be the case that an ad hoc institution is set up or more frequently that the sector specific agency also supervises the project.

3. Bundling and ownership

The large variety of PPPs and the complexity of such agreements give the possibility to study numerous contractual issues.

Attention has been devoted for example to the question of how to finance an infrastructure (Caillaud and Tirole, 2004), how to allocate risk transfer (Martimort and Zantman, 2006) and which the implication of contract flexibility are (Bajari and Tadelis, 2001; Ellman, 2006).

In this section we focus of the issue of bundling or separation (unbundling) of different phases of a project. In particular, we consider the relation with the type of ownership.

This choice is driven by the aim of giving the main theoretical insights of the literature related to the analysis we present in chapter 2, where we study the incentive of a railway provider to invest in rolling stock according to the ownership of the assets.
3.1. **Bundling or separation of phases**

As we illustrated in the previous section, a project is composed of different stages.

Consider for example the realization of an infrastructure as a high-speed railway line. First in the design phase both main principles and all the details of how to build tracks and wagons are established.

Then, the building of all components takes place and the service is finally available.

To let the travellers using it, it is necessary implement a reservation and a payment system. Of course, all the procedure related to maintenance and safety must be applied and monitored.

Even if this was a simplified description of the process, it is easy to understand the general complexity of realizing projects of this kind and to think about the specific professional skills that each phase requires.

Therefore, one of the main issues to consider when defining a project is to choose between bundling and unbundling: in order to achieve a certain objective, which is the best organizational form? Bundle activities together and let one firm doing everything or contract separately with different firms?

Moreover, the analysis of the advantages and disadvantages of bundling can be combined with the analysis of the role of the ownership. Recall that in a public-private partnership, all the tasks are bundled and performed by a private firm or consortium of firms.

We can identify two streams in the literature according to the approach used that is an incomplete or complete contract approach.

### 3.1.1. **Incomplete contract approach**

As for the incomplete contract approach, we mainly consider to Hart (2003) and Bennett and Iossa (2006). Both these paper refer to the property rights literature à la Grossman and Hart (1986).

The main result of these papers is that when the partner building and/or running a facility can undertake an effort that affects quality and/or costs, there will be distortions with respects to the first best. The distortions depend on the impossibility to write ex ante a contract, so that negotiation ex post takes place.
Moreover, the existence of a positive externality between stages is what (or at least mainly) calls for bundling activities. The intuition for this result is that if the tasks are undertaken by the same firm, then the positive effect of the effort can be internalized.

If there is a negative externality (see Bennett and Iossa, 2006), then the results are less clear-cut.

In what follows, we see in more details the frameworks used to analyze this issue and highlights when bundling is to be preferred and the role of the ownership. The latter point has been deepened by Bennett and Iossa. In fact, while in Hart unbundling corresponds to public ownership and bundling to private ownership, Bennett and Iossa consider private ownership by the builder, by the management firm and by a consortium of firms.

Hart (2003) studies the case of the private provision of a public service and the effect on social benefits of two different types of arrangements, bundling and unbundling of building and running the facility.

The model is of the type of Hart, Schleifer and Vishny (1997), where the provider of the service can be either the government or a private contractor and it is also the owner of the facility. The manager can undertake an investment in improving quality or an investment that reduces both cost and quality.

Because of the distortions with respect to first best level, private ownership is to be preferred when the reduction in quality is small or when cost reduction is unlikely and the public manager has low incentives (i.e. his gains from the activity is low).

In a manner similar to Hart Schleifer and Vishny, Hart develops a model in three stages: first the government decides who to contract with, then the facility is built and finally the service is operated.

When building the facility, two types of unverifiable investments can be chosen: a productive investment $i$ that makes the facility more attractive and easier to run and/or an unproductive investment $e$ that reduces total costs but also quality. Both types of investment affect social benefits generated by the provision of the service: $i$ increases benefits and $e$ decreases them.

The government can choose between contracting separately with a builder and a managing firm or bundling the activities of building and operating the facility (PPP).
The comparison of the two choices leads to underline the existence of a trade-off when choosing between bundling and unbundling.

*Under unbundling, neither the social benefits nor the effect on operating cost are internalized by the builder.*

*The appropriate level of unproductive investment is achieved. On the other hand, he is investing less in the productive activity, relative to the social optimum.*

*Under PPP, the social benefits are still not internalized but now the builder internalizes the operating costs and chooses a larger amount both of the unproductive and of the productive investment.*

Bennett and Iossa (2006) focus on a specific kind of agreement that is private finance initiative (PFI), in which a consortium of private firms is in charge of all the phases from designing to operating the project. This type of arrangement is usually valid for a very long period of time (25-30 years) and the government defines output specifications, so that the public sector specifies the basic standards of the project and then the private partner decides how to satisfy them.

The framework is quite similar to Hart, but they assume that investments are non contractible ex ante and verifiable ex post.

The initial contract specifies some basic standard requirements and their satisfaction is observable and verifiable.

At the beginning of each stage, the firm can decide to invest in some innovation and such investment has the previously said characteristics, non contractible ex ante and verifiable ex post.

The two types of investment $a$ and $e$ positively affect the social benefits: $a$ can be done at the beginning of the building stage and affects the costs of that stage as well as the cost in the management stage and the residual value; $e$ can be undertaken at the beginning of the management stage and has an impact only on management costs.

Only the owner of the facility during the contract period has the power to decide whether any given innovative activity can be implemented or not, so that Nash bargaining occurs between the owner and the firm willing to undertake the investment.

A very interesting point is that the investment at the building stage $a$ can have either a positive or a negative externality across stages.
With a positive externality, bundling is always optimal regardless of the ownership.

This result is due to the ability of bundling to internalize the positive externality across stages.

The choice of the type of ownership to induce the optimal level of investments depends on the effect of $a$ on managing costs and on the residual value, compared with the effect on social benefits: if social benefits are larger, then PFI is optimal for the investment at the building stage. Similarly, PFI is optimal for the investment at the managing stage if the benefits of the innovation dominate social benefits.

Consider now these results if the investments were not verifiable as in Hart: bargaining never occurs and each party simply cares about maximizing its own profit.

Under private ownership, investments in the building stage only occur under PPP and when the builder is the owner of the facility. On the other hand, investments in the management stage only occur under PPP and when the firm operating the service owns the facility.

Under public ownership investments never take place. Therefore, even if investments are not verifiable, bundling is still optimal and PFI dominates each other arrangement.

The key component of Bennett and Iossa result is the sign of the externality: as there exists a positive externality, the aim is to internalize its effects. But in case of a negative externality, the result no longer holds.

In case of a not too strong negative externality, the optimal level for $a$ is reached through unbundling and in particular the ownership should be public if the marginal social benefit is larger than the marginal effect on the residual value. Vice versa, ownership should be assigned to the builder.

Clear cut results cannot be obtained if the negative externality is strong. Under both PFI and traditional procurement there is a problem of overinvestment. Therefore, the choice between bundling and unbundling depends on the relative sizes of the effects on welfare.

An important aspect of the contractual agreement is the treatment of the residual value of the assets when the contract expires. Different clauses can be included in the contract and state who owns the facility at the end of the management stage. Therefore, an important issue is to
understand the role of a change in the ownership when the contract expires.

One of the most common provisions is the automatic transfer of ownership to the public sector. Bennett and Iossa show that under PFI, this condition is welfare reducing if there is a positive externality, as the anticipation of the transfer of ownership at the end of the contract diminishes the incentive to invest, while if there is an option for negotiation, then the case for PFI is strengthened.

3.1.2. Complete contract approach

The matter of bundling different tasks together when there is moral hazard (see Holmström and Milgrom, 1990 and 1991) suggests that a positive externality across stages leads to the choice of bundling the activities, while a negative externality calls for a separation of the activities.

As for the specific issues of PPP in a moral hazard environment we mainly refer to Martimort and Pouyet (2008), Iossa and Martimort (2008) and for an application to the transport sector to Iossa and Martimort (2009).

Similar to the incomplete contract approach, also in the complete contract approach the incentive to invest is different according to the choice of bundling or splitting tasks. In particular, this effect is strictly related to the type of externality that the investment has on costs and/or quality. Moreover, bundling is optimal if there exists a positive externality.

In Martimort and Pouyet (2008) firms can exert a non-verifiable effort, which affects the quality of the project (for example, an infrastructure) and thereby the operating costs. They show that what drives the choice of the organizational structure is the sign of the externality: if the effort decreases the costs, i.e. with a positive externality, when all the phases are performed by the consortium, the moral hazard problem is alleviated.

Moreover, it appears that bundling almost always dominates unbundling. In particular under public ownership bundling and unbundling are equivalent with a negative externality, but bundling dominates with a positive externality.

If the ownership is private, then bundling strictly dominates unbundling if there is a positive externality the private value of infrastructure is low.
A general framework for the analyses of public-private partnerships is given by Iossa and Martimort (2008).

They consider the relationship between the public sector and a risk averse agent, which can choose unobservable efforts reducing costs and enhancing the quality of the project. Among the extensions of the model, they consider the issue of bundling of stages.

*From a welfare point of view, they show that bundling leads to higher welfare then separation of stages if the externality of efforts is positive. If the externality is negative, there is no difference between the two arrangements.*

They also show that *if the infrastructure has a residual value at the end of the contract, then PPP strictly welfare dominates traditional contracting if and only if the externality between the design and the operation phases is positive.*

Based on this model, Iossa and Martimort (2009) analyze when PPPs provide adequate incentives to the private providers to invest in the improvement of quality in the transport sector.

Suppose that demand is stochastic and depends on firm efforts in lowering costs and increasing the quality of the infrastructure.

The ability of the consortium to internalize the effect of the quality enhancing investment grants gains in welfare. *Total welfare under bundling is strictly larger than under unbundling and the positive gap increases with the externality due to the quality of the infrastructure.*

The bundle of activities shifts more risk to the consortium and gives it more incentive to invest in quality enhancing, explaining why PPP projects are characterized by more risk transfer than traditional procurement.

As Iossa and Martimort underline, the advantage of a public-private partnership is stronger if a better quality of the infrastructure significantly benefits the operational phase and when the demand is stable, as for example in the transport sector (at least in the short run).

They also consider the role of contract length and underline that it should be longer when demand risk is lower and capital investment is greater, as for example in the railway industry.

In the next chapter we tackle the issue of bundling and ownership, applied to transport sector, in particular to the rolling stock market in the railway industry. A transportation authority willing to allow more
competitors to take part into the selection could decide to provide the operator with rolling stock, which appears to be a source of disadvantage for potential competitors.

The decision about bundling or unbundling, corresponding respectively to private and public ownership of the rolling stock is taken by a regulator that has to select the provider of a service when a firm is already active and another firm compete for the market.

Differently from Iossa and Martimort we consider an adverse selection problem over the cost parameter of firms. Moreover, there is only one type of investment undertaken by the operator that reduces costs and positively affects the value of rolling stock.

It appears that the choice of the ownership in order to obtain the higher level of investment is mainly related to the its effects on rolling stock and costs and to the cost of public funds.

4. Theory and empirical evidence

As seen above, according to Hart (2003) the advantage of choosing one type of provision against the other depends on the characteristics of the service at glance.

The argument in favour of unbundling is stronger the more difficult to specify the quality of the service.

If the quality of the facility can be well specified, whereas the quality of the service cannot, then traditional provision seems more appropriate since there is less concern about the underinvestment in $i$ under unbundling than about overinvestment in $e$ under bundling.

In contrast, PPP is more appropriate if the quality of the service can be well specified in the initial contract, whereas the quality of the building cannot be: underinvestment in $i$ in traditional procurement may be a serious issue, while overinvestment in $e$ under PPP is not.

This result seems coherent with the evidence of some projects and could also be applied to interpret the applicability of PPP to core and non-core services (Riess, 2005). Consider two examples: hospitals and schools. In the case of hospitals, the clinical treatments and the clinical support services (like laboratories) are core services and all other services (such as cleaning) can be considered non-core activities. In the case of schools, the core service is the education provided by the
teachers and all the “accommodation” service are non-core. What can be expected is that when applying traditional procurement to clinical or school core-service better results are obtained, than under PPP as the outcome of the provision of services can’t be well specified.

Evidence from U.K. (Bennett and Iossa, 2006) is that PFI did not work so well for schools and hospitals and the above interpretation may (at least partially) explain it.

Bennett and Iossa also show that the results obtained in their research are consistent with empirical evidence in the U.K.: average estimated savings of PFI projects are larger then savings in traditional procurement. For some sectors is also evident the existence of positive externalities across stages, which were exploited by PFI. For example, under PFI prisons with innovative design also show a reduced cost of running them; highway projects include long-term initiatives when building, for improving the service in term of noise reduction. Instead, for other projects which involve continuous adaptation, as IT provision, public ownership may be preferred.

As for PPP in the transport sector, investments in safety of rail and air transportation can be interpreted as investment \( a \) in Bennett and Iossa model (investment at the building stage that could have either a positive or a negative externality): some innovations about safety are not foreseeable when the contract is signed and once investments are undertaken at the building stage, they also unavoidably affect the management stage. More safety enhances public utility but also raises costs of maintaining tracks and all the structures needed. If the raising in costs is not too high, then according to the previous results, unbundling eases the problem of underinvestment and the choice of the ownership depends on the magnitude of the effect on residual value compared with the social benefit. But if the investment in safety implies very high increase in operating costs, then the argument in favour of bundling or unbundling and type of ownership is not conclusive.
Chapter 2

The Role of Public or Private Ownership and the Incentive to Invest in the Railway Industry

1. Introduction

In recent years the transportation sector has foregone important changes. The European Union is strongly keen on completing the internal market for transport and most of the changes in the sector are due to its deregulation interventions.

At the end of Nineties\(^3\), the Member States laid down the strategic principles to revitalise the railway sector, to increase its competitiveness and attractiveness with customers.

In passenger transport, the approach proposed by the European Commission consists in open access for international services and in regulated competition, for instance, for urban services through the tendering of franchises or public service contracts (competition for the tracks). Moreover, a separation between infrastructure management and transport service provision is required: this separation must be done to ensure at least non-discriminatory network access, such as capacity allocation and setting of track access charges.

Different types of organization were adopted across countries (Seabright et al., 2003) also depending on the type of service, i.e. long distance v. local passenger service.

For example, in Germany, the Länder created agencies, with the aim of managing the procurement of local passenger rail transport. Each agency can freely choose the provider of the service: for example, it can negotiate directly with the incumbent without involving other competing firms or can announce a tender. As a result of the introduction of competition for the market, Deutsche Bahn competitors increased substantially their market share (Lalive and Schmutzler, 2008).

\(^3\) White Paper on Rail Transport, 1996.
It is probably too early to have a precise view of the effect of the deregulation, but according to Friebel et al (2008) the reform has positively affected the level of technical efficiency of the market for passenger transportation in Europe, increasing the productivity trend by 0.5% per year on average.

Even if one of the aim of these reforms is to induce the creation of a competitive environment, in which many firms compete for the market, in most cases the transportation service is still provided by the historical incumbent.

A first question that can be raised immediately is why, even if there exists competition for the market, the (local) railroad market is mainly dominated by the presence of the historical incumbent.

An entrant willing to provide transportation service does not have an easy access to the market of rolling stock, which is an essential input to provide the service. Rolling stock have special features that do not allow firms to obtain them easily; this is mainly due to the specificity of the demand, the absence of a secondary market and the long production time.

The technical and operational characteristics of rolling stock within the country and its specificity for certain routes results in limited interchange ability between different types of stocks.

The absence of a secondary market does not allow firms operating (or willing to operate) the service to obtain good quality rolling stock, partly because of the previous point and partly because of the qualitative standards required, which are not always achievable if the rolling stock are old. Finally, the time needed for the production is usually long, between two and four years.

Different solutions have been attempted to solve this problem. The most famous is probably the British system, where the Rolling Stock Leasing Companies (ROSCOs) lease the rolling stock material to the companies in charge of the provision of the service (Train Operating Companies, TOCs). The ROSCOs were created with the privatization of British Rail in 1996, when its rolling stock were distributed to three specialized companies, with the aim of promoting competition both among the TOCs and among the ROSCOs.

In Sweden, the local Public Transport Authorities are responsible for setting up the auction and decide which operator will provide the
service. The rolling stock material is supplied by AB Transitio, a company founded in 1998, whose shareholders are the six Public Transit Authorities, which are under local public ownership.

In Italy, the rolling stock are almost entirely owned by Trenitalia. The national antitrust authority recognized\textsuperscript{4} the difficulties in obtaining the rolling stock, but it decided not to define it as an “essential facility”, because of the lack of non-duplicability at reasonable social cost. The proposed solution is to allow the selected supplier to begin with the provision of the service after it has obtained the necessary assets or the acquisition of the rolling stock by the local authority.

If the local authority auctioning off the service buys the rolling stock, it ensures that a company not owning the assets can take part into the selection too and reduces the advantage of the incumbent.

The issue we focus on is what the effect of the ownership of the rolling stock is on the incentive of the provider of the service to invest.

The model has two periods. In the first one, the incumbent is the only firm in the market and he has its own rolling stock. In the second period, the local authority selects the provider.

We consider two different frameworks: \textit{private ownership} and \textit{public ownership}, where the expression \textit{private} or \textit{public} refers to who is the owner of the rolling stock in the second period. As suggested, a possible solution to the problem of the rolling stock is that the local authority supplies the provider of the service with the assets and we define this framework as \textit{public ownership}. On the other hand, if the firm has to be equipped with its own assets, then it operates in the \textit{private ownership} framework.

The provider of the service undertakes an investment that reduces its costs and increases the value of the rolling stock. This is an important feature, because the firm cannot decide to affect only one of the two aspects (i.e. only the costs or only the value of the rolling stock) as we will explain in detail afterwards.

\textsuperscript{4} AGCM AS262 “Reperimento del materiale rotabile ferroviario necessario per l’espletamento delle gare per l’aggiudicazione dei servizi ferroviari di competenza regionale”, 26/06/2003. An Italian region asked the antitrust authority an opinion on how to get rolling stock for a procurement process.
The difference between the two frameworks is not only who the owner of the rolling stock is, but also how the investment affects the assets and the social utility.

We compare the levels of investment and study the probability of the incumbent to remain active in the second period.

Under complete information, the cost of public funds and the transferable part of the investment play an important role in determining the gap between the investment under private ownership and under public ownership. In the second period together with the cost of public funds it is necessary to consider the increase in social utility under public ownership.

If there is a problem of asymmetric information over costs, then the local authority grants a rent to the provider in order to incentive the truthful revelation of its cost parameter and this effect could overcome the difference between the levels of investments in the two frameworks. Therefore, a priori it is not possible to define in which context the level of investment is higher.

Moreover, we consider the probability for the incumbent to remain active in the second period and it appears that the investment made in the first period gives him an advantage also at the selection stage, because the choice of the authority also accounts for the effect of the incumbent’s first period investment.

This paper is close to the analysis of Bennett and Iossa (2006), Hart (2003) and Laffont and Tirole (1993)\(^5\).

The first paper considers the choice between private or public ownership and bundling or not of the different stages to build and manage a facility. Differently from their analysis, we assume that there is a selection among firms and that it is affected by an asymmetric information problem over the cost parameter of the firms. Besides, we do not have negotiation between parties, as when the authority decides to buy the rolling stock, it makes a take-it-or-leave-it offer to the incumbent.

Our approach is closer to Laffont and Tirole, even if we have only an adverse selection problem and no moral hazard. We consider a firm that can undertake an investment, but here the effect of it on the firms’ and on the regulator’s utility is more composite. In this respect, the effect of our investment is similar to what illustrated by Bennett and Iossa. But while Bennett and Iossa assume that the effect of the investment on the payoffs is always the same, regardless of the kind of arrangement, here the effect depends on the framework in which the firm operates. Part of the investment is assumed to be transferable together with the use of the rolling stock: under private ownership it will always be exploited only by the incumbent if it is active in the second period, but under public ownership it could be exploited by the competitor if it is selected.

The paper is organized as follows: after a brief review of the literature, section 3 presents the baseline model to illustrate the framework before the reform that is when only the incumbent operates the transportation service. Section 4 studies two possible scenarios in which the firm can operate, that is under private ownership or under public ownership.

The models under private and under public ownership are developed in section 5 with complete information and in section 6 with asymmetric information.

Section 7 characterizes the optimal rule chosen by the authority for the selection of the provider in both frameworks when there is asymmetric information.

Section 8 concludes and offers possible extensions to the analysis.

2. Literature review

The introduction of the selection among different providers and the cooperation between the private and the public sector are related to the issue of the public-private partnerships (PPPs).

The partnership between the private and the public sector has been used as an alternative to traditional procurement in many countries and in many sectors.

In the U.K. and in Portugal, PPPs are really a significant source of provision of infrastructure and service: in the U.K. these types of arrangements are used for rail or road projects, in the building and
managing of schools, hospitals and prisons, while in Portugal the partnerships are almost restricted to the road sector.

In this paper we compare two frameworks. The first one is labelled _private ownership_ framework, and it can be interpreted as a period of concession followed by a PPP agreement, because there is a selection among firms providing the service with their own assets. The second one is the _public ownership_ framework, where at the end of the first period there is an automatic transfer to the public authority of the assets and in the second period a selection among firms takes place. The selected supplier provides the service but use the public assets, so that there is a traditional provision of the public service.

We mainly refer to the work of Hart (2003), of Bennett and Iossa (2006) and of Laffont and Tirole (1993).

Both the work by Hart and by Bennett and Iossa are developed in an incomplete contract framework: Hart assumes that investments are never verifiable and Bennett and Iossa assume that there exists negotiation between the parties\(^6\).

Hart (2003) considers the case where building a facility and managing it can be contracted out together (a situation defined as “bundling”, or PPP) or separately (“unbundling”). In the building stage, two types of unverifiable investments can be undertaken: a productive investment that makes the facility more attractive and easier to run (“good” investment) and an unproductive investment that saves on total costs but reduces quality (“bad” investment). The comparison between bundling and unbundling leads to underline the existence of a trade-off between internalization of benefits and costs and the level of investments. Under unbundling, neither the benefits nor the operating costs are internalized by the builder, so that the optimal level of unproductive investment is achieved, but there is an underinvestment problem in the “good” investment. Under PPP, the benefits are not internalized but now the builder internalizes the costs and chooses a larger amount both of the unproductive and of the productive investment.

Bennett and Iossa (2006) consider the case of Private Finance Initiative (PFI), a form of PPP in which typically all the phases from designing to

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\(^6\) They show that they would achieve the same results under Hart assumption
operating the project are contracted out to a consortium of private firms for a long period of time (usually 25-30 years) and the government defines output specifications.

There are two types of investment that positively affect the social benefits. One type can be undertaken at the beginning of the building stage: it reduces the costs of that stage, raises the residual value of the assets and reduces (positive externality) or increases (negative externality) the cost in the management stage. Another type of investment can be done at the beginning of the management phase and reduces only the cost of that stage.

Assuming the residual value of the facility to be independent of its use when the contract expires, the bundling of the two stages is always optimal in the case of a positive externality (and in the case of a not too strong negative externality), because it induces the internalization of the externality. The choice between public and private ownership to achieve the optimal level of investment in each period, depends on whether the effect on costs and residual value are larger than the effect on social benefit: if it is the case, then PFI is optimal.

When there is a strong negative externality, i.e. when the investment in the building stage increases the cost in the managing stage, then the results are not clear as both under PFI and traditional procurement there is a problem of overinvestment. Therefore, the choice between bundling and unbundling depends on the relative sizes of the effects on welfare.

Differently from Bennett and Iossa we introduce a selection procedure for the second period provider and an adverse selection problem. The local authority offers the firms a menu of contracts among which to choose, in order to overcome the asymmetric information over cost. Moreover, at the end of the first period the authority may choose a new partner. If the local authority decides to acquire the incumbent’s rolling stock (public ownership framework), then it makes a take-it-or-leave-it offer to the incumbent that partially depends on the level of investment undertaking during the first period. Therefore, differently from Bennett and Iossa, there is no negotiation between the parties over the benefits of the investment.

The introduction of the selection and the asymmetric information problem makes our analysis similar to Laffont and Tirole (1993, ch. 8), who develop a two-period model in a complete contract framework.
They consider an incumbent who operates as a monopolist in a market and in the second period it has to compete with another firm for the market.

In a specification of the general model, the monopolist can undertake an effort – *learning by doing*. The effort reduces the costs of the same period in which it is undertaken and the first period effort of the incumbent reduces the second period costs if he is active and can also lower the competitor’s second period costs in case she is selected by the regulator.

Laffont and Tirole also study the choice of the optimal “breakout” rule that is the cost threshold defined by the regulator to select the second period provider: if the entrant’s cost is lower than this threshold, she will be preferred.

In the model by Laffont and Tirole the effort has an impact only on the cost function of the firms, while in our model, the investment lowers only the incumbent’s costs and affects the competitor’s payoff only through the usage of the rolling stock in the public ownership case.

In contrast to Hart and to Bennett and Iossa, in the present paper there is only one type of investment. It acts along two dimensions as it reduces the costs and increases the value of the rolling stock, which looks quite similar to Bennett and Iossa investment at the building stage with positive externality. However, in our case the magnitude of the effect is strictly related to the framework in which the firm operates. As we describe in the next sections, it is crucial to distinguish the effect on the rolling stock under *private ownership* from the effect under *public ownership*, because while in the first case only the incumbent can take advantage of his investment, in the latter case part of the benefit can be exploited by the second period provider, regardless of which firm is selected.

An important characteristic of the kind of investment we are considering is that it has two simultaneous effects so that the firm cannot decide to strategically affect only one dimension that is it is not able to only lower the costs or only increase value of the rolling stock.

### 3. The baseline model: two-periods monopoly

Let us now illustrate investment decisions in the two different frameworks that we define as *private ownership* and *public ownership*. 
In order to set the basic characteristic of the model, we begin with considering first the case of a local authority in charge of regulating the transportation service on a certain route. There exists only a monopolist \( I \) that operates a fixed amount of rail transport for two periods and receives a transfer.

By providing the service, the incumbent invests in order to better organize his activity on that route. Therefore, we assume that in both stages, the incumbent can undertake an investment that has a double effect: it lowers the incumbent’s costs and increases the value of his rolling stock. Note that there are two positive simultaneous effects and the firm cannot choose to affect only the costs or the assets side. In this baseline model, this aspect is not particularly significant, but as we will stress in the following sections, when the incumbent has to compete for the market, then it emerges that having operated the line, the incumbent has an advantage due to past investment\(^7\).

Consider for example the incumbent that invests in order to find a better way to organize the workforce, so that the cost of labour dedicated to the maintenance is reduced both in the first period and in the second one.

Moreover, thank to a better organisation, the maintenance is more effective and the quality of the rolling stock is increased.

In the following period, there still exists a positive effect on the quality, that we define as partly transferable and partly non transferable (or specific). In fact, a share of the investment is strictly linked to the assets – for example, seat covers are cleaner and less damaged, so that do not need to be renewed – and is therefore transferable together with the rolling stock. On the other hand, another part is specific or non transferable, so that only the incumbent can take advantage of it – for example, operating the line gives the incumbent the possibility to understand how to better organize cleaning service.

The effect on the assets in the same period in which the investment takes place represents the advantage for the firm operating the line. His experience and knowledge will have an impact on the selection rule of the authority, as we will see in the next paragraphs.

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\(^7\) The effects of the investment on the incumbent’s and the competitor’s payoff are different according to the type of ownership chosen by the authority.
The incumbent’s costs of providing the service depend on his cost parameter $\beta^l$ and on the positive effect $a^l_1$ of the investment he undertakes at time $t$. In the first period the costs are:

$$C^l_1(\beta^l, a^l_1) = \beta^l - a^l_1$$

and in the second period are

$$C^l_2(\beta^l, a^l_1, a^l_2) = \beta^l - ka^l_1 - a^l_2,$$

where $k$ is the part of the effect of the first period investment that also lowers second period costs. We assume that $k \in (0,1)$, because the effect in the second stage is lower then in the first one.

The investment is costly and his disutility at time $t$ is $d(a^l_1)^8$.

The effect of the investment on the rolling stock in each period is represented by $b(a^l_1)^9$, which we define direct effect.

The incumbent also benefits of an indirect effect, that is an increase in the second period of the value of the assets by $(g + h)b(a^l_1)$, where $g$ is the transferable part and $h$ is the specific or non transferable part. The indirect effect is smaller then the direct effect, so that $(g + h) \in (0,1)$.

As we are now considering the baseline model, the incumbent fully internalizes the effect of the first period investment. In general, the user of the rolling stock in the second period can take advantage of $gb(a^l_1)$, but only the incumbent can exploit $hb(a^l_1)$.

The local authority provides the incumbent with $F^l_i(\beta^l)$ in addition to the incumbent’s costs $C^l_i$, so that the total amount of transfers is

$$T^l_i = F^l_i + C^l_i.$$

The monopolist maximizes his payoff $P^l = P^l_1 + \delta w^l P^l_2$, in which $\delta$ is the discount factor and $w^l$ is the probability that the incumbent provides the service in the second period, which in this baseline model is equal to one.

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8. $d' > 0$, $d'' > 0$
9. $b' > 0$, $b'' < 0$, $d' > b'$
In detail, the incumbent maximizes:

\[ P^I = T_1^I - C_1^I (\beta^I, a_1^I) - d(a_1^I) + b(a_1^I) + \]

\[ + \delta w^I \left[ T_2^I - C_2^I (\beta^I, a_1^I, a_2^I) - d(a_2^I) + (g + h)b(a_1^I) + b(a_2^I) \right] \]

under the participation constraint \( P^I \geq 0 \).

The provision of the transportation service generates a positive social value \( S \), such that it is always socially desirable to provide the service.

The local authority maximizes the following welfare function:

\[ W = S(1 + \delta) + P^I - (1 + \lambda)\left(T_1^I + \delta T_2^I\right) \]

where \( \lambda > 0 \) is the cost of public funds.

### 3.1. Optimal levels of investment

Under complete information, the local authority knows the cost parameter \( \beta^I \) and observes the costs in each period \( C^I \). The maximization of the welfare function subject to the participation constraint of the incumbent \( P^I \geq 0 \) determines the optimal levels of investment:

\[ a_1^I : \quad d'(a_1^I) = 1 + b'(a_1^I) + \delta v^I \left[ k + (g + h)b'(a_1^I) \right] \quad (1) \]

\[ a_2^I : \quad d'(a_2^I) = 1 + b'(a_2^I) \quad (2) \]

In each period, the optimal level of investment is set such that the marginal cost of the investment is equal to the marginal benefit of cost savings and of the increase in the value of the rolling stock.

In the first period, the optimal level accounts for the expected marginal benefits in the second period of reducing the costs and of the increase in the value of the rolling stock due to the indirect effect (recall that here \( w^I = 1 \)).

The expected gain from the second period has a positive effect on the choice of first period investment, so that the higher the savings in second period costs and/or the higher the indirect effect on the value of the rolling stock, the higher the level of optimal investment.

Finally, notice that the level of investment positively depends on the probability of providing the service in the next period: the higher the
probability, the higher the expected benefits the incumbent is able to internalize.

4. Regulation and competition: the selection of the second period supplier

According to the European Directives, the regional rail transport has been opened up to competition and regions should issue public tenders in order to choose the provider of local rail services for a period of exclusive franchising.

It is interesting to understand what happens to the investments if the incumbent knows that in the next period the authority will choose the provider of the service according to a certain selection procedure.

Consider a two period model: in the first period, the incumbent is in charge of the transportation service and receives transfers in return; the monopolist knows that at the beginning of the second period a provider is selected by the local authority.

In the first period, the incumbent uses his own rolling stock, while in the second period there are two possible frameworks: the *private* (PR) or the *public* (PUB) ownership framework.

The concept of *private* or *public* refers to the ownership of the rolling stock in the second period: if the second period provider uses its own assets, then it operates in the *private ownership* framework; if the local authority provides the firm with the assets, then the second period provider operates in the *public ownership* framework.

For simplicity, we consider an initial stage (*t=0*), where the local authority announces in which framework the firm will operate in the second period and gives information about the selection.

The local authority decides if the transportation service in the second period is provided using private or public assets and offers a menu of contract from which the incumbent can choose at the beginning of the first period. Moreover, it defines the breakout rule, that is the rule according to which the second period supplier is chosen.

If the local authority opts for the *private ownership*, then the second period provider must be equipped with its own assets. Note that a firm can take part to the selection at no cost, but will buy or will adapt the
rolling stock to the need of the specific routes - if necessary - only once it is chosen\(^\text{10}\).

If the local authority opts for the \textit{public ownership}, then it buys the rolling stock of the incumbent at the end of the first period and lends them to the second period provider.

The incumbent \(I\) and the competitor \(C\) are characterized by the cost parameter \(\beta^I - \beta^I\) for the incumbent and \(\beta^C\) for the competitor – which are independently drawn from the same distribution \(f(\beta)\) on \([\beta, \bar{\beta}]\).

In order to choose the firm that grants the highest level of welfare, the local authority has to define a threshold for the cost parameter below which it will prefer to switch from the incumbent to the competitor. We call this threshold “breakout rule” and we denote it as \(\beta_j^{**}\), where \(j=\text{PR, PUB}\) according to the framework, i.e. \textit{private} or \textit{public}. The threshold is calculated to maximizes the welfare function and the competitor is selected if her cost parameter \(\beta^C < \beta_j^{**}\).

In the first period \((t=1)\), the incumbent provides the service: the costs he bears, the investment he undertakes and the effect on next period costs and on the rolling stock are (qualitatively) the same as in the baseline model.

Under public ownership, the local authority pays a price (that will be defined later) to the incumbent to obtain the rolling stock at the end of the first period.

At the beginning of the second stage \((t=2)\), the local authority chooses the provider according to its threshold \(\beta_j^{**}\).

In the public ownership framework, the authority rents the assets to the selected firm; under private ownership, the provider uses its own rolling stock, so that if it is not already equipped, it has to acquire the assets.

We assume that if a firm is not selected, it obtains zero profit. The authority is in charge of the service only at the local level, so that it is

\(^{10}\) Recall that it takes time to both buy or adapt the rolling stock to the route.
interested only in maximizing the “local” welfare and considers the outside option for the firms equal to zero.

In the second period, only one out of the two firms is selected. Similarly to what explained in the baseline model, the firm active in the second period can undertake an investment that positively affects both its own costs and the value of the rolling stock.

If the incumbent is active in the second period, then he bears costs

\[ C_{2,j}^I(\beta^I, a_{1,j}^I, a_{2,j}^I) = \beta^I - ka_{1,j}^I - a_{2,j}^I. \]

while if the competitor is active, her costs are:

\[ C_{2,j}^C(\beta^C, a_{2,j}^C) = \beta^C - a_{2,j}^C. \]

The cost functions of the two firms are different, because the incumbent enjoys the positive effect of the first period investment, so that his costs are decreased by \( ka_{1,j}^I \).

As for the effect on the rolling stock, each firm is able to internalize it in different ways, according to the kind of ownership under which the provider operates. We describe how this effect has an impact on the utility function of the firms in the next sections.

Finally, if the ownership is public, the local authority compensates the provider with a reward that depends on the level of investment it undertakes.

The timing of the game can be summarized as follows:

We analyze the incentive to invest under private and under public ownership, both in the case of complete information and of asymmetric information.
5. Selection under complete information

Under complete information, the authority observes the costs at the end of the period and also knows the cost parameter of each firm.

5.1. Private ownership of the assets

In the framework with private ownership, the incumbent operates the service in the first period and at the beginning of the second period the local authority selects a provider, that can be either the incumbent $I$ or the competitor $C$.

In particular, we consider the case of the incumbent facing a competitor, which does not own the suitable rolling stock and has therefore to bear a higher fix cost than the incumbent to obtain the appropriate assets, if she is selected.

As we mentioned in the introduction, a difficulty in the supply of the assets is due to the specificity of the demand, so that it is not unusual to have the provider spending on the modification of the rolling stock.\(^{11}\)

In general, this “extra-cost” is a measure of the disadvantage of the competitor in comparison with the incumbent. We assume that the incumbent fix costs are zero and for the entrant are $R_0$.

The incumbent undertakes the investment in the first period and he reduces the costs and increases the value of the rolling stock. The effect on the second period costs and value of the assets is realized and internalized only by the incumbent, if he remains active.

In the second period, the provider can make the investment and obtain again positive effects on costs and assets.

As we noticed in the baseline model, the investment in the first period has a double effect and the firm cannot choose to affect only the value of the rolling stock or only next period costs. Therefore, we are implicitly assuming that the incumbent has an advantage over the competitor due to past investment.

The following table summarizes the effect of the investments on the value of the rolling stock for the firms and for the local authority:

\(^{11}\) The higher cost could also be interpreted as the difference between the annual cost of the rolling stock for the competitor and for the incumbent.
Under private ownership, the hypothesis about the double effect of investment is not particularly relevant, because the incumbent knows that he could be the only one who gains from it. But under public ownership, the rolling stock are available to the second period provider and the competitor could also benefit from the investment, in case the incumbent is not chosen by the authority.

The incumbent maximizes his expected payoff \( P_{PR}^I = P_{1,PR}^I + \delta w_{PR}^I P_{2,PR}^I \), where \( \delta \) is the discount factor and \( w_{PR}^I \) is the probability that the incumbent remains active in the second period operating with his own assets:

\[
P_{PR}^I = T_{1,PR}^I - C_{1,PR}^I (\beta^I, a_{1,PR}^I) - d(a_{1,PR}^I) + b(a_{1,PR}^I) + \delta w_{PR}^I [T_{2,PR}^I + - C_{2,PR}^I (\beta^I, a_{1,PR}^I, a_{2,PR}^I) - d(a_{2,PR}^I) + (g + h)b(a_{1,PR}^I) + b(a_{2,PR}^I)]
\]

(1.1)

The participation constraint is \( P_{PR}^I \geq 0 \) (1.2)

The competitor’s payoff, if she is active in the second period is:

\[
P_{PR}^C = T_{2,PR}^C - C_{2,PR}^C (\beta^C, a_{2,PR}^C) - d(a_{2,PR}^C) + b(a_{2,PR}^C) - R_0
\]

(1.3)

and the participation constraint is \( P_{PR}^C \geq 0 \) (1.4)

Under complete information, the regulator maximizes the sum of net consumers surplus and firms payoffs:

\[
W_{PR} = S(1 + \delta) - (1 + \lambda)[T_{1,PR}^I + \delta T_{2,PR}^I] + P_{1,PR}^I + \delta P_{2,PR}^I
\]

with \( i = I, C \)

subject to (1.2) and (1.4) binding:
where $\beta_{PR}^{**}$ is the breakout rule under private ownership, i.e. the threshold that determines the second period provider.

Under complete information, the authority chooses the breakout rule maximizing (1.5) with respect to $\beta_{PR}^{**}$. Therefore, the competitor is chosen if her cost parameter is lower than $\beta_{PR}^{**}$ such that:

$$\beta_{PR}^{**} + R_0 \leq \beta^l - ka_{1,PR} - (g + h) b(a_{1,PR}).$$

Not surprisingly, the authority wants to choose the firm whose cost parameter leads to the lower costs and it accounts for the advantages that the incumbent obtains from the first period investment.

The maximization program of the authority becomes:

$$S(1 + \delta) - (1 + \lambda) \left( \beta_{PR}^l - a_{1,PR}^l + d(a_{1,PR}^l) - b(a_{1,PR}^l) \right) +$$

$$- \delta(1 + \lambda) \left[ \left( 1 - F(\beta_{PR}^{**}) \right) \beta^l - a_{2,PR}^l - ka_{1,PR} + d(a_{2,PR}^l) + (g + h) b(a_{1,PR}^l) \right] +$$

$$+ \int_{\beta}^{\beta_{PR}^{**}} \left( \beta^c - a_{2,PR}^c + d(a_{2,PR}^c) - b(a_{2,PR}^c) + R_0 \right) f(\beta^c) d\beta^c \right\}$$

Therefore, the socially optimal levels of investment are:

$$a_{1,PR}^l : \quad d'(a_{1,PR}^l) = 1 + b'(a_{1,PR}^l) + \delta \left[ 1 - F(\beta_{PR}^{**}) \right] k + (g + h) b'(a_{1,PR}^l) \right] \quad (1.7)$$

$$a_{2,PR}^l : \quad d'(a_{2,PR}^l) = 1 + b'(a_{2,PR}^l) \quad (1.8)$$

$$a_{2,PR}^c : \quad d'(a_{2,PR}^c) = 1 + b'(a_{2,PR}^c) \quad (1.9)$$

In the complete information framework, the optimal levels of investment are set, such that in each period the provider of the service
equates the marginal cost of the investment to its marginal benefit in terms of cost savings and of the increase in the value of the rolling stock.

In the first period, the optimal level of the investment also accounts for the expected gain from the investment in the case of the incumbent remaining active in the second period: the marginal benefit of reducing the cost in the second period and the marginal increase in the value of the rolling stock due to first period investment.

Comparing this results to those obtained in the baseline model (i.e. (1) vs. (1.7) and (2) vs. (1.8) and (1.9)), we obtain the following result:

**Proposition 1.A.** Under private ownership of the assets, the introduction of the selection lowers the incentive to invest in the first period: the lower the probability to be active in the second period, the lower the investment.

This result is due to the uncertainty over the fact that the incumbent will be active in the second period, so that he could not be able to internalize entirely the effects of his investment. The choice of the second period investment is not affected.

5.2. Public ownership of the assets

We analyze the case in which the authority decides to buy the rolling stock of the incumbent. In the first period, the service is operated by the monopolist using his own assets. The incumbent knows that at the end of the first stage, the assets are transferred to the local authority, who pays a price – here normalized to zero – plus a reward for the investment undertaken that positively affects the value of the rolling stock.

In the second period, the selected firm provides the service using public rolling stock. The payoff of the not selected firm is zero.

Consider now the effect of the investment for each party, i.e. the incumbent, the competitor and the local authority.

As for the decrease in costs, nothing changes relative to the baseline model or under private ownership: the firm invests and lowers the costs and increases the value of rolling stock in the same period and also generates an effect in the next stage.
The role of the investment on the rolling stock is different in some respects than we described in the previous frameworks.

If the authority decides to acquire the assets (public ownership), it pays a price to the incumbent at the end of the first period: a fixed part, normalized to zero, plus a reward which depends on the effect of the investment on the rolling stock $x(a_{1,PUB}^I)^{12}$. We assume that the authority offers $x(a_{1,PUB}^I) \leq b(a_{1,PUB}^I)$, so that the incumbent cannot obtain more than under private ownership.

Moreover, at the end of the second period, the authority pays a reward $x(a_{2,PUB}^I)$ to the provider $i$ of the service to compensate it for the investment that affects the value of the public rolling stock.

As the authority can supply the provider with rolling stock, it creates a potentially more competitive environment in the next periods, because also firms not owning the assets at all can compete for the market. In order to take into account this benefit, we consider an increase in the social value of the service of $u(a_{2,PUB}^i)^{13}$, i.e. consumers benefit of the investment in the public owned rolling stock.

Under public ownership, the incumbent obtains $x(a_{1,PUB}^I)$ in the first period and if he remains active, he enjoys entirely the indirect effect $(g + h)b(a_{1,PUB}^I)$ and is paid $x(a_{2,PUB}^I)$ for the second period investment.

If the competitor is selected, then she benefits of the transferable part $gb(a_{1,PUB}^I)$, because she uses the public assets and receives the reward $x(a_{2,PUB}^C)$, if she invests.

The following table summarizes the effect of the investment on the value of the rolling stock for the firms and for the authority:

---

12 $x' > 0$, $x' < 0$. The model does not include negotiation between parties: $x$ can be understood as a take it or leave it offer.

13 $u' > 0$, $u' < 0$
The objective function of the authority is:

\[
W_{PUB} = S(1 + \delta) + P_{1,PUB}^l - (1 + \lambda)T_{1,PUB}^l + \left[ b(a_{1,PUB}^l) - (1 + \lambda)x(a_{1,PUB}^l) \right] + \\
+ \delta P_{2,PUB}^l - \delta(1 + \lambda)T_{2,PUB}^l + \left[ b(a_{2,PUB}^l) - (1 + \lambda)x(a_{2,PUB}^l) \right] + u(a_{2,PUB}^l)
\]

The incumbent maximizes his expected payoff

\[
P_{PUB}^l = P_{1,PUB}^l + \delta w_{PUB}^l P_{2,PUB}^l,
\]

where \( w_{PUB}^l \) is the probability that the incumbent remains active in the second period and operates with public assets:

\[
P_{PUB}^l = T_{1,PUB}^l - C_{1,PUB}^l (\beta^l, a_{1,PUB}^l) - d(a_{1,PUB}^l) + x(a_{1,PUB}^l) + \delta w_{PUB}^l \left[ T_{2,PUB}^l + \\
- C_{2,PUB}^l (\beta^l, a_{2,PUB}^l, a_{2,PUB}^l) - d(a_{2,PUB}^l) + (g + h)b(a_{1,PUB}^l) + x(a_{2,PUB}^l) \right]
\]

and the participation constraint is \( P_{PUB}^l \geq 0 \) \hspace{1cm} (2.1)

The competitor’s payoff if she is active in the second period is:

\[
P_{PUB}^C = T_{2,PUB}^c - C_{2,PUB}^c (\beta^c, a_{2,PUB}^c) - d(a_{2,PUB}^c) + gb(a_{1,PUB}^l) + x(a_{2,PUB}^c)
\]

and the participation constraint is \( P_{PUB}^C \geq 0 \) \hspace{1cm} (2.3)

Under complete information, the regulator maximizes the objective function subject to (2.2) and (2.4):
\[ W_{PUB} = S(1 + \delta) - (1 + \lambda)\left(C_{1,PUB}^I(\beta^I, a_{1,PUB}^I) + F_{1,PUB}^I\right) + \\
+ \left[(b(a_{1,PUB}^I) - (1 + \lambda)x(a_{1,PUB}^I)\right] + \\
- \delta(1 - F(\beta_{PUB}^{**}))(1 + \lambda)\left(C_{2,PUB}^I(\beta^I, a_{1,PUB}^I, a_{2,PUB}^I) + F_{2,PUB}^I\right) + \\
+ \delta(1 - F(\beta_{PUB}^{**}))\left[b(a_{2,PUB}^I) - (1 + \lambda)x(a_{2,PUB}^I) + u(a_{2,PUB}^I)\right] + \\
(2.5) \\
- \delta \int_{\beta}^{\beta_{PUB}^{**}} \left[(1 + \lambda)\left(C_{2,PUB}^C(\beta^C, a_{2,PUB}^C) + F_{2,PUB}^C\right)f(\beta^C)d\beta^C + \\
+ \delta \int_{\beta}^{\beta_{PUB}^{**}} \left[b(a_{2,PUB}^C) - (1 + \lambda)x(a_{2,PUB}^C) + u(a_{2,PUB}^C)\right]f(\beta^C)d\beta^C \\
\right]
\]

where \( \beta_{PUB}^{**} \) is the breakout rule under public ownership, i.e. the threshold that determines the second period provider.

Under complete information, the authority chooses the (optimal) breakout rule maximizing (2.5) with respect to \( \beta_{PUB}^{**} \). Therefore, the competitor is selected if her cost parameter is lower than \( \beta_{PUB}^{**} \), such that:
\[ \beta_{PUB}^{**} \leq \beta^I - ka_{1,PUB}^I - hb(a_{1,PUB}^I). \]

As we noticed in the private ownership framework, the local authority selects the firm whose cost parameter grants the lower level of costs and also accounts for the advantage that only the incumbent can enjoy.

Hence, the program becomes:
\[ S(1 + \delta) - (1 + \lambda)\left[\beta^I - a_{1,PUB}^I + d(a_{1,PUB}^I)\right] + b(a_{1,PUB}^I) + \\
- \delta(1 - F(\beta_{PUB}^{**}))(1 + \lambda)\left[\beta^I - a_{2,PUB}^I - ka_{1,PUB}^I + d(a_{2,PUB}^I) - (g + h)b(a_{1,PUB}^I)\right] + \\
+ \delta(1 - F(\beta_{PUB}^{**}))\left[b(a_{2,PUB}^I) + u(a_{2,PUB}^I)\right] + \\
(2.6) \\
- \delta \int_{\beta}^{\beta_{PUB}^{**}} \left[(1 + \lambda)\left(\beta^C - a_{2,PUB}^C + d(a_{2,PUB}^C) - gb(a_{1,PUB}^I)\right)f(\beta^C)d\beta^C + \\
+ \delta \int_{\beta}^{\beta_{PUB}^{**}} \left[b(a_{2,PUB}^C) + u(a_{2,PUB}^C)\right]f(\beta^C)d\beta^C
\]
The optimal levels of investment are:

\[ a_{1,PUB}^I : \]
\[ d'(a_{1,PUB}^I) = 1 + \frac{1}{1 + \lambda} b'(a_{1,PUB}^I) + \delta g b'(a_{1,PUB}^I) + \delta (1 - F(\beta_{PUB}^I)) k + h b'(a_{1,PUB}^I) \] \hspace{1cm} (2.7)

When the authority acquires the rolling stock, the optimal level of the investment is such that the marginal cost equals the marginal benefit of cost savings and of increased quality. The direct effect of the investment on the rolling stock is discounted for the value of public funds, because in this framework the authority grants a reward to the incumbent undertaking the investment.

The investment in the first period affects the second period utility of the provider, so the authority also considers the benefits in the second stage. Part of the them is certain and it corresponds to the transferable part: it is strictly linked to the rolling stock and the second period provider benefits of it, regardless of who the provider is. Part of the benefits is considered in expectation: the reduction in the second period cost and the non transferable effect on rolling stock is realized only if the incumbent keep on being the monopolist.

\[ a_{2,PUB}^I : \]
\[ d'(a_{2,PUB}^I) = 1 + \frac{1}{1 + \lambda} [b'(a_{2,PUB}^I) + u'(a_{2,PUB}^I)] \] \hspace{1cm} (2.8)

\[ a_{2,PUB}^C : \]
\[ d'(a_{2,PUB}^C) = 1 + \frac{1}{1 + \lambda} [b'(a_{2,PUB}^C) + u'(a_{2,PUB}^C)] \] \hspace{1cm} (2.9)

In the second period, the optimal levels of investment are determined by the equality of the marginal cost to the marginal benefit of cost savings and of the increase in the value of assets and in social utility. As in the first period, the authority accounts for the cost of public funds.

Compare these results with the baseline results ((1) v. (2.7) and (2) v. (2.8) and (2.9)):

**Proposition 1.B.** Under public ownership, the selection lowers first period investment. This effect is larger, the lower the probability for the incumbent to remain active and the larger the cost of public funds.
In the second period, if

\[ b'(a^i_2) > \frac{1}{1 + \lambda} \left[ b'(a^i_{2,PUB}) + u'(a^i_{2,PUB}) \right] \]

then the level of investment with the selection is lower. This is more likely to hold, the larger the cost of public funds and the smaller the marginal increase in the social utility.

In the first period, the investment is lower because part of the benefits is not certain but is only expected. Moreover, the local authority requires a lower level if the cost of public funds is high, because it has to pay for the rolling stock a price which is partly related to the investment.

In the second period, the public ownership generates an additional social utility and if it is high enough, it could overwhelm the effect of the costs of public funds and lead to higher investment.

5.3. Ownership and investment levels under complete information

After the introduction of the selection, the uncertainty over the possibility of being active in the second period lowers the optimal level of investment in the first period both under private and under public ownership. Moreover, under public ownership the cost of public funds plays an important role on the choice of investment, because the larger the cost, the higher the probability to have lower investment in both periods, even if the social utility increases thank to the possibility of supplying next period competitor with the rolling stock.

Let us now compare what happens to the level of investment once the competitive selection is enforced, according to the type of ownership (i.e. consider the first order conditions (1.7) v. (2.7), (1.8) v. (2.8) and (1.9) v. (2.9)):

**Proposition 1.C.** In the first period, the investment under private ownership is larger than under public ownership, the larger the cost of public funds and the probability for the incumbent to be selected and the smaller the transferable part of the investment.

In the second period, the investment under private ownership may be larger or smaller than under public ownership. The
investment is larger under private ownerships if the cost of public funds is high and the marginal effect of public ownership on the social utility is small.

Under public ownership, the local authority acquires the incumbent’s rolling stock and pays in both periods a reward which is function of the effect of the investment. Therefore, the higher the cost of public funds, the lower is the incentive of the authority to promote the investment.

The gap between the private and public investment decreases (increases) the larger (the smaller) the transferable part of the investment. Recall that under public ownership the second period provider enjoys the transferable part of the investment and this is true regardless of who the provider is. If this part is very significant, i.e. if \( g \) is large, then the local authority wants the incumbent to invest more because it is sure that the second period provider will benefit of it (vice versa for low value of \( g \)).

Finally, in the second period some ambiguity emerges. If the additional social utility generated by the public ownership of the assets does not overcome the effect of the cost of public funds, then the investment required is still lower than in the private ownership framework.

6. Selection under asymmetric information

Let us now assume that the local authority does not know the cost parameter of each firm \( \beta^i \). It only knows that the two cost parameters are independently drawn from the same distribution \( f(\beta) \) over the interval \( [\beta, \bar{\beta}] \) with cumulative distribution \( F(\beta) \). The local authority offers a menu of contract \( \{F^i, C^i\} \) among which each firm can choose\(^\text{14}\).

\(^{14}\) For notational simplicity, we do not always substitute to \( a^I_1 \) the expression \( \beta^I - C^I_{1,PR}(\beta^I) \) and similarly for \( a^I_2 \).
6.1. Private ownership

Under asymmetric information, the authority has to take into account both the incentive and the individual rationality constraints of the incumbent.

The incentive constraint represents the decrease in the rent for the incumbent when the cost parameter is marginally increased:

\[
\dot{\pi}^I_{PR}(\beta^I) = - \left[ d'\left(\beta^I - C^I_{1,PR} (\beta^I)\right) - b'\left(\beta^I - C^I_{1,PR} (\beta^I)\right) \right] + \\
- \delta (1 - F(\beta^*_{PR})) (1 - k) \left[ d'\left(\beta^I - C^I_{2,PR} (\beta^I) - k\alpha^I_{1,PR}\right) \right] + \\
- b'\left(\beta^I - C^I_{2,PR} (\beta^I) - k\alpha^I_{1,PR}\right) + \delta (1 - F(\beta^*_{PR})) (g + h) b'\left(\beta^I - C^I_{1,PR} (\beta^I)\right) 
\]  

(3.1)

The participation constraint ensures that all type of firm can take part into the selection. The following participation constraint states that the least efficient type gets no rent at the optimum:

\[
P^I_{PR}(\bar{\beta}^I_{PR}) = 0 
\]  

(3.2)

The same constraints have to be considered for the competitor. The incentives constraint is:

\[
\dot{\pi}^C_{PR}(\beta^C) = - d'\left(\beta^C - C^C_{2,PR} (\beta^C)\right) + b'\left(\beta^C - C^C_{2,PR} (\beta^C)\right) 
\]  

(3.3)

The participation constraint is such that the most inefficient type does not obtain a rent.

\[
P^C_{PR}(\beta^*_{PR}) (\beta^I) = 0 
\]  

(3.4)

The most inefficient competitor is the one with cost parameter \(\beta^*_{PR}\), that is the highest value of the cost parameter that the local authority is willing to accept.

Hence, the authority’s maximization program is:
\[\int_{\beta}^\beta \{s(1 + \delta) - (1 + \lambda)\left[C_{1,PR} + d(a^l_{1,PR}) - b(a^l_{1,PR})\right] - \lambda P^l_{PR} + \]

\[-\delta\{1 - F(\beta^*_C)\}(1 + \lambda)\left[C^l_{2,PR} + d(a^l_{2,PR}) - (g + h)b(a^l_{2,PR}) - b(a^l_{2,PR})\right] + \]

\[-\delta\int_{\beta}^\beta \left(1 + \lambda\right)(C^C_{2,PR} + d(a^C_{2,PR}) - b(a^C_{2,PR}) + R_0) + \]

\[+ \lambda \hat{P}^C_{PR} \{f(\beta^C)d\beta^C \} \}

subject to constraints from (3.1) to (3.4)

The optimal levels of investment are:

\[a^l_{1,PR} : \]

\[d'(a^l_{1,PR}) = 1 + b'(a^l_{1,PR}) + \delta\{1 - F(\beta^*_C)\}[k + (g + h)b'(a^l_{1,PR})] + \frac{\lambda}{1 + \lambda} \frac{F(\beta^l)}{f(\beta^l)} \left[ d''(a^l_{1,PR}) - b''(a^l_{1,PR}) - \delta(1 - F(\beta^*_C))(g + h)b''(a^l_{1,PR}) \right] \]

\[a^l_{2,PR} : \]

\[d'(a^l_{2,PR}) = 1 + b'(a^l_{2,PR}) - \frac{\lambda}{1 + \lambda} \frac{F(\beta^l)}{f(\beta^l)} (1 - k) \left[ d''(a^l_{2,PR}) - b''(a^l_{2,PR}) \right] \]

\[a^C_{2,PR} : \]

\[d'(a^C_{2,PR}) = 1 + b'(a^C_{2,PR}) - \frac{\lambda}{1 + \lambda} \frac{F(\beta^C)}{f(\beta^C)} \left[ d''(a^C_{2,PR}) - b''(a^C_{2,PR}) \right] \]

The same observations made in the case of complete information apply, but here the local authority considers the distortion from asymmetric information. Compare the results with those obtained under complete information: the investment required to an inefficient type is always distorted downwards and only if the provider is the most efficient, than the level of the investment is the first best solution.

It is interesting to notice that in the second period, even if the incumbent and the competitor were equally efficient, the optimal level of investment for the incumbent is higher as his distortion is lower due to the first period investment.
6.2. Public ownership

As in the previous case, the authority accounts for the incentive and individual rationality constraints of the firms.

The incentive constraint of the incumbent is:

\[
\dot{P}_{\text{PUB}}^I(\beta^I) = -d^I(\beta^I - C^I_{1,\text{PUB}}(\beta^I)) + x^I(\beta^I - C^I_{1,\text{PUB}}(\beta^I)) + \\
- \delta(1-k)[(1-F(\beta^*_{\text{PUB}}))d^I(\beta^I - C^I_{2,\text{PUB}}(\beta^I) - ka^I_{1,\text{PUB}})] + \\
- x^I(\beta^I - C^I_{2,\text{PUB}}(\beta^I) - ka^I_{1,\text{PUB}})] + \delta(1-F(\beta^*_{\text{PUB}}))(g+h)b^I(\beta^I - C^I_{1,\text{PUB}}(\beta^I))
\]

4.1

The individual participation constraints is written, so that no rents are left to the most inefficient type:

\[
P_{\text{PUB}}^I(\overline{\beta}^I) = 0
\]

4.2

The incentive and individual rationality constraint for the competitor are respectively:

\[
\dot{P}_{\text{PUB}}^C(\beta^C) = -d^C(\beta^C - C^C_{2,\text{PUB}}(\beta^C)) + x^C(\beta^C - C^C_{2,\text{PUB}}(\beta^C))
\]

4.3

\[
P_{\text{PUB}}^C(\beta^*_{\text{PUB}}(\beta^I)|\beta^I) = 0
\]

4.4

The authority’s maximization program becomes:

\[
\int_{\beta^I} \left[ S(1+\delta) - (1+\lambda)(C^I_{1,\text{PUB}} + d(a^I_{1,\text{PUB}}) - x(a^I_{1,\text{PUB}})) - \lambda P_{\text{PUB}}^I + \\
- \delta(1-F(\beta^*_{\text{PUB}}))\delta(1+\lambda)(C^I_{2,\text{PUB}} + d(a^I_{2,\text{PUB}}) - (g+h)b(a^I_{1,\text{PUB}}) - x(a^I_{2,\text{PUB}})) + \\
- \delta(1+F(\beta^*_{\text{PUB}}))\delta(1+\lambda)(C^C_{2,\text{PUB}} + d(a^C_{2,\text{PUB}}) - gb(a^I_{1,\text{PUB}})) + \lambda P_{\text{PUB}}^C + \\
+ [b(a^I_{1,\text{PUB}}) - (1+\lambda)x(a^I_{1,\text{PUB}})] + \delta(1-F(\beta^*_{\text{PUB}}))\delta(a^I_{2,\text{PUB}}) - (1+\lambda)x(a^C_{2,\text{PUB}})] + \\
+ \delta(1+F(\beta^*_{\text{PUB}}))\delta(1+\lambda)x(a^I_{2,\text{PUB}})] + \\
+ \delta(1-F(\beta^*_{\text{PUB}}))u(a_{2,\text{PUB}}) + \int_{\beta}^\beta u(a^C_{2,\text{PUB}}) f(\beta^C) d\beta^C \right] f(\beta^I) d\beta^I
\]

4.5
subject to constraints from (4.1) to (4.4).

The optimal levels of the investment are such that

\[ a_{1,\text{PUB}}^* : \]
\[ d'(a_{1,\text{PUB}}^*) = 1 + \frac{1}{1 + \lambda} b'(a_{1,\text{PUB}}^*) + \delta g b'(a_{1,\text{PUB}}^*) + \delta \left[ 1 - F(\beta_{\text{PUB}}^*) \right] k + h b'(a_{1,\text{PUB}}^*) + \]
\[ - \frac{\lambda}{1 + \lambda} \frac{F(\beta')}{f(\beta')} \left[ d''(a_{1,\text{PUB}}^*) - x''(a_{1,\text{PUB}}^*) \right] - \delta \left[ 1 - F(\beta_{\text{PUB}}^*) \right] (g + h) b''(a_{1,\text{PUB}}^*) \]

in the first period and in the second period:

\[ a_{2,\text{PUB}}^* : \]
\[ d'(a_{2,\text{PUB}}^*) = 1 + \frac{1}{1 + \lambda} \left( b'(a_{2,\text{PUB}}^*) + u'(a_{2,\text{PUB}}^*) \right) + \]
\[ - \frac{\lambda}{1 + \lambda} \frac{F(\beta')}{f(\beta')} (1 - k) \left[ d''(a_{2,\text{PUB}}^*) - x''(a_{2,\text{PUB}}^*) \right] \]

\[ a_{2,\text{PUB}}^* : \]
\[ d'(a_{2,\text{PUB}}^*) = 1 + \frac{1}{1 + \lambda} \left( b'(a_{2,\text{PUB}}^*) + u'(a_{2,\text{PUB}}^*) \right) + \]
\[ - \frac{\lambda}{1 + \lambda} \frac{F(\beta^c)}{f(\beta^c)} \left[ d''(a_{2,\text{PUB}}^*) - x''(a_{2,\text{PUB}}^*) \right] \]

As we noticed in the private ownership framework, asymmetric information distorts the level of investment downwards, except for the most efficient firm, while the level of investment of the incumbent is higher than the one exerted by the competitor.

In the next section we compare the levels of investment in the two frameworks.

6.3. Ownership and investment levels under asymmetric information

Let us compare the levels of investment under private and under public ownership in case of asymmetric information. We have already seen that – absent asymmetric information – in the first period the public ownership regime requires lower investment, while for the second
period investment some ambiguities arise. Here, we have an additional trade-off between the effect of the investment and the rent from asymmetric information.

Consider the first order conditions that define the levels of investment (from (3.6) to (3.8) and from (4.6) to (4.8)) and in particular the right hand side.

Let’s first analyze the total marginal benefit (i.e. everything in the right hand side of first order conditions) except the rent from asymmetric information. The optimal investment of the incumbent is higher under private ownership than under public ownership for similar reasons as under complete information, i.e. for high value of the cost of public funds and of the probability for the incumbent to remain active and for low marginal increase in the additional social utility.

As for the rent effect, we notice that in both periods, the distortion due to the asymmetric information under private ownership is larger than under public ownership as we assumed that the reward is at most equal to the value that the firm assigns to the investment. The smaller the reward, the lower the distortion, the higher the level of investment.

Therefore, even if due to the effect on costs and rolling stock the level of investment with private ownership were larger than with public ownership, the difference between the rent under public ownership and the rent under private ownership could prevail and overturn the result.

Therefore, in comparing the level of investment in the two frameworks, both the reward and the increase in the social utility are fundamental in determining when the investment is higher.

Thus, we can summarize:

**Proposition 2.** Under asymmetric information, it is not possible to define which ownership leads to higher level of investment, because it depends on whether the effect on costs and rolling stock or the distortion due to asymmetric information prevails.

If the first is stronger, then the investment is higher under private ownership, otherwise if the distortion is larger, the investment is higher under public ownership.
Therefore, it is not possible to obtain clear-cut results: there is a trade-off between the effect of the investment and the distortion from asymmetric information, which determines in which framework the investment is higher.

7. Choosing the second period supplier

In the previous section, we have compared the levels of investment considering the probability for the incumbent to be the second period provider as given, but the criterion of the selection (breakout rule) is endogenously defined. The local authority determines the maximum cost parameter $\beta_j^* \text{ that can characterize the competitor selected as second period supplier.}$

Recall that the higher $\beta_j^*$, the lower the probability for the incumbent to be active in the second period.

Understanding how the local authority chooses the rule is important because it is a tool that the authority can use to incentive investment in the first period: the higher the probability to be active in the second period, the more the incumbent is induced to invest even if he will not benefit of the effect on second period costs or of the indirect effect on the rolling stock.

The local authority defines the breakout rule that maximizes its utility function, that is it defines for which level of the cost parameter it is willing to switch from the incumbent to the competitor: if the competitor has a cost parameter at most equal to the one defined by the rule, then she is selected as the second period supplier for the transportation service.

In what follows, we want to explore under which conditions the incumbent remains active in the second period, because as mentioned in the introduction, this is the most frequent case across countries.

We also characterize the breakout rule under private and under public ownership in case of asymmetric information over the cost parameter of the firms: we study if the incumbent or the competitor is favoured. The competitor is favoured if the threshold is higher than under complete information and vice versa, the incumbent is favoured if the breakout rule is lower than under complete information.
In order to carry out the analysis, we specify the model and observe the effect in the change of the different parameters on the threshold.

### 7.1. Choice criteria under private ownership

The threshold $\beta_{PR}^*$ is the value of the cost parameter defined by the breakout rule under private ownership with asymmetric information. It is determined by the local authority, maximizing its welfare function (3.5) with respect to $\beta_{PR}^*$.

The competitor is selected if the following inequality holds:

$$
\left[ C_{2,PR}^I + d(a_{2,PR}^I) - (g + h)b(a_{2,PR}^I) - b(a_{2,PR}^I) \right] + \frac{\lambda}{1 + \lambda} \frac{F(\beta_{PR}^I)}{f(\beta_{PR}^I)} \left[ (1 - k) \left( d'(a_{2,PR}^I) - b'(a_{2,PR}^I) \right) - (g + h) b'(a_{2,PR}^I) \right] > 0
$$

(5.1)

According to the cost parameter of the incumbent, what is the probability of him to keep on being the second period supplier? And does asymmetric information favours the incumbent or the competitor? One firm is favoured under asymmetric information if it has greater probability to be active in the second period than under complete information. The incumbent is favoured if $\beta_{PR}^* < \beta_{PR}^{**}$, while the competitor is favoured if $\beta_{PR}^* > \beta_{PR}^{**}$, where $\beta_{PR}^{**}$ is the threshold under complete information.

Recall that $\beta_{PR}^{**}$ is such that $\beta_{PR}^{**} + R_0 \leq \beta^I - ka_{1,PR}^I - (g + h)b(a_{1,PR}^I)$.

In order to perform the comparison, it is convenient to specify the model as follows: we assume that the cost parameters of the firms are uniformly distributed over the interval $[1,2]$.

Moreover, the disutility of the investment is $d(a_{i,PR}^I) = (a_{i,PR}^I)^2$ and the benefit on the rolling stock is $b(a_{i,PR}^I) = \frac{1}{5} a_{i,PR}^I$. 

50
We use some graphs to illustrate the probability for the incumbent to be selected and to show when the breakout rule favours the incumbent and when it favours the competitor\(^\text{15}\).

The graphs marked by A. represent for each value of the incumbent’s cost parameter \(\beta^I\) (on the horizontal axis) the probability to be selected, according to different values of a certain parameter.

The figure denoted by B. represents for each value of \(\beta^I\) (on the horizontal axis) the value of \(\Delta_{PR}\), defined as the difference between the threshold under asymmetric information \(\beta_{PR}^*\) and the one under complete information \(\beta_{PR}^{**}\): 

\[
\Delta_{PR} = \beta_{PR}^* - \beta_{PR}^{**}.
\]

If \(\Delta_{PR}\) is positive, then the competitor is favoured by asymmetric information (\(\beta_{PR}^* > \beta_{PR}^{**}\)), while if it is negative (\(\beta_{PR}^* < \beta_{PR}^{**}\)), then the incumbent is favoured.

Keeping constant all other parameters, we examine how \(\Delta_{PR}\) changes when the following parameter changes:

“Extra” cost of the competitor

The selected competitor has to bear an extra cost relative to the incumbent. The larger this cost, the lower the breakout rule that the authority chooses at the beginning of the game, so that the incumbent has more probability to be the second period provider (Figure 1.A.). When the extra cost is large, the authority is willing to change the provider of the service only if it is very efficient.

Figure 1.B. represents the difference between the cost threshold defined under asymmetric and under complete information. If the competitor has a cost parameter equal to the threshold, then she is favoured under asymmetric information.

Consider for example when the extra cost is equal to 0.2, then if the cost parameter of the incumbent \(\beta^I \leq 1.3\) the threshold is so low that the competitor is not selected. For \(\beta^I > 1.3\), the breakout rule allows a competitor to enter and the asymmetric information favours her.

\(^{15}\) The numerical value of some of the simulations are in the appendix A.1.
because under complete information the authority would choose a lower threshold, that is it would lower the probability of the competitor to enter.

Figure 1.A.: probability for I to be active in t=2, according to $\beta^I$ for different values of the extra cost of the competitor ($R_0$)

Figure 1.B.: effect of the extra cost of the competitor ($R_0$) on $\Delta_{PR}$, according to $\beta^I$
**Remark 1.A.**: The larger the extra-cost of the competitor, the larger the probability for the incumbent to provide the service in the second period. If the competitor is efficient enough, then she is favoured under asymmetric information.

**Cost of public funds:**

The increase in the cost of public funds induces the authority to lower the probability for the incumbent to be selected (Figure 2.A.).

The cost of public funds affects the rent from asymmetric information in both periods: the higher the cost, the higher the rent.

The probability to be selected as second period provider has a double effect on the choice of first period investment: it raises the level of investment through the total marginal benefit but it decreases it through the rent. In this specification of the model, the second effect seems to prevail so that the local authority prefers to lower the probability for the incumbent to be active to mitigate the rent effect.

**Figure 2.A.**: probability for I to be active in t=2, according to β’ for different values of the cost of public funds (λ)

If the competitor has a cost parameter “low enough”, the larger the cost of public funds, the more the competitor is favoured. (Figure 2.B.)
Figure 2.B.: effect of the cost of public funds ($\lambda$) on $\Delta_{PR}$, according to $\beta^I$

**Remark 2.A.:** The larger the cost of public funds, the lower the probability for the incumbent to be active in the second period. Under asymmetric information, the competitor is favoured.

**Discount factor:**
If the discount factor increases, the probability to keep the incumbent in the second period is larger (Figure 3.A.).

Once the competitor is efficient enough, then the breakout rule under asymmetric information favours the competitor (Figure 3.B.).

**Remark 3.A.:** The larger the discount factor, the higher the probability to keep the incumbent. Under asymmetric information, the competitor is favoured.
Figure 3.A.: probability for I to be active in t=2, according to $\beta^I$ for different values of the discount factor ($\delta$)

Figure 3.B.: effect of the discount factor ($\delta$) on $\Delta_{PR}$, according to $\beta^I$

Transferable and non transferable part of the investment:

The increase in the indirect effect – either in the transferable or in the non transferable part – reduces the cost parameter chosen as threshold in the breakout rule (Figure 4.A.).
Figure 4.A.: probability for I to be active in t=2, according to $\beta^I$ for different values of the indirect effect ($g + h$)

The competitor is favoured by the asymmetric information:

Figure 4.B.: effect of the indirect effect ($g + h$) on $\Delta_{PR}$, according to $\beta^I$

**Remark 4.A.** the larger the indirect effect, the larger the probability for the incumbent to provide the service in the second period. Asymmetric information favours the competitor.
The larger the indirect effect, the larger the advantage that only the incumbent can internalize in the second period. Therefore, the authority prefers not to switch the provider, unless the competitor is very efficient.

**Reduction in second period costs:**

Let us now consider the effect of first period investment on the second period costs. As for the indirect effect on the rolling stock, the authority is willing to change the service provider only if the entrant is very efficient, because the incumbent is able to save on costs in the second period. (Figure 5.A.)

![Figure 5.A.: probability for I to be active in t=2, according to β\(^t\) for different values of the reduction in second period costs due to first period investment (k)](image)

Asymmetric information, does not always favour the competitor (Figure 5.B.).
Figure 5.B.: effect of the reduction in second period costs due to first period investment \((k)\) on \(\Delta_{PR}\), according to \(\beta^I\)

*Remark 5.A.:* the incumbent has higher probability to provide the service, the larger the effect of the first period investment on the second period costs.

*Under asymmetric information, it is more probable to have the competitor favoured.*

We have obtained these results for a particular specification of the model. Nevertheless, we can observe that under private ownership the first period investment grants to the incumbent an advantage in the second period selection. The incumbent seems to have higher probability to remain the monopolist of the local market in the second period, the higher the effect of his investment on the costs and on the rolling stock.

Under private ownership, the local authority does not own the assets so that it cannot benefit directly of the investment and does not obtain an increase in the social utility as it happens under public ownership. Therefore, the authority is more willing to favour the firm that is more efficient, and this efficiency is measured also including the effect on costs and rolling stock.
Only the cost of public funds plays the opposite role, as it lowers the probability of the incumbent to continue to be the monopolist.

The local authority is ready to entrust the competitor with the service only if she has a very low cost parameter. The competitor has higher probability to become the second period provider under asymmetric information, than under complete information.

### 7.2. Choice criteria under public ownership

As for the private ownership framework, we determine the optimal breakout rule maximizing the local authority program (4.5) with respect to \( \beta_{\text{PUB}}^* \). The competitor is selected if the following disequality holds:

\[
\left[ C_{2, \text{PUB}}^i + d(a_{2, \text{PUB}}^i) - \lambda b(a_{1, \text{PUB}}^i) \right] - \frac{b(a_{2, \text{PUB}}^i) + u(a_{2, \text{PUB}}^i)}{1+\lambda} + \\
\left[ \lambda \frac{F(\beta_{\text{PUB}}^i)}{f(\beta_{\text{PUB}}^i)} [(1-k)(d'(a_{2, \text{PUB}}^i) - x'(a_{2, \text{PUB}}^i)) - (g+h)b'(a_{1, \text{PUB}}^i)] \right] >
\]

\[
\text{(5.2)}
\]

\[
\left[ C_{2, \text{PUB}}^c + d(a_{2, \text{PUB}}^c) \right] - \frac{b(a_{2, \text{PUB}}^c) + u(a_{2, \text{PUB}}^c)}{1+\lambda} + \\
\left[ \frac{\lambda}{1+\lambda} \left( d'(a_{2, \text{PUB}}^c) - x'(a_{2, \text{PUB}}^c) \right) \frac{F(\beta_{\text{PUB}}^c)}{f(\beta_{\text{PUB}}^c)} \right]
\]

Following the same line of the analysis of the breakout rule under private ownership, we specify the model for the public ownership framework.

The cost parameter that characterizes each firm is independently drawn from a uniform distribution in the interval [1,2].

The disutility of the effect of the investment is \( d(a_{i, \text{PUB}}^i) = (a_{i, \text{PUB}}^i)^2 \) and the benefit on the rolling stock is \( b(a_{i, \text{PUB}}^i) = \frac{1}{5} a_{i, \text{PUB}}^i \).

As the ownership is public, the local authority pays a reward

\[
x(a_{i, \text{PUB}}^i) = \Theta b(a_{i, \text{PUB}}^i)
\]
and gains $u(a^i_{2,PUB}) = \omega b(a^i_{2,PUB})$, where $\theta, \omega \in [0,1]$.

As before, we use some graphs to illustrate the probability for the incumbent to be selected and to show when the breakout rule favours the incumbent and when it favours the competitor\(^{16}\).

The graphs defined by A. report for each value of the incumbent’s cost parameter $\beta^I$ (on the horizontal axis) the probability to be selected, according to different values of the parameter at hand.

The graphs B. illustrate the value of $\Delta_{PUB}$, for any given $\beta^I$ (on the horizontal axis) and $\Delta_{PUB}$ is the difference between the threshold under asymmetric information $\beta^*_{PUB}$ and the threshold under complete information $\beta^{**}_{PUB}$. $\beta^*_{PUB}$ is such that $\beta^*_{PUB} \leq \beta^I - ka^I_{1,PUB} - hb(a^I_{1,PUB})$.

Therefore, $\Delta_{PUB} = \beta^*_{PUB} - \beta^{**}_{PUB}$. If $\Delta_{PUB}$ is positive, then the competitor is favoured ($\beta^*_{PUB} > \beta^{**}_{PUB}$), while if it is negative, then the incumbent is favoured ($\beta^*_{PUB} < \beta^{**}_{PUB}$).

We keep constant all other parameter and examine the effect of the variation of the following parameter:

**Cost of public funds**

The larger the cost of public funds, the larger the rent from asymmetric information; therefore, the local authority lowers the probability for the incumbent to remain active (Figure 6.A.).

If the cost of public funds increases, the probability of having a breakout rule that under asymmetric information favours the competitor increases (Figure 6.B.). In contrast to what seen for the private ownership, above a certain level of the cost parameter, the threshold favours the incumbent.

**Remark 1.B:** the larger the cost of public funds, the lower the probability for the incumbent to be the second period provider.

\(^{16}\) The numerical value of some of the simulations are in the appendix A.1.
The probability that the breakout rule favours the competitor is higher, the higher the cost of public funds.

Figure 6.A.: probability for I to be active in t=2, according to $\beta^I$ for different values of the cost of public funds ($\lambda$) on $\Delta_{PUB}$ according to $\beta^I$

Figure 6.B.: effect of the cost of public funds ($\lambda$) on $\Delta_{PUB}$ according to $\beta^I$
Discount factor

The increase in the value of the discount factor leads the authority to choose a lower threshold, so that the incumbent has more probability to be selected (Figure 7.A.). The competitor is more often favoured when the discount factor increases (Figure 7.B.).

Figure 7.A.: probability for I to be active in t=2, according to $\beta^i$ for different values of the discount factor ($\delta$)

Figure 7.B.: effect of the discount factor ($\delta$) on $\Delta_{PUB}$, according to $\beta^i$
**Remark 2.B:** the probability to select the incumbent is higher for high values of the discount factor.

The breakout rule favours the competitor more often for high discount factor.

**Indirect effect of the investment on the rolling stock**

Under public ownership, regardless of who is the provider in the second period, the firm benefits of the transferable part $g$, while only if the incumbent remains active he enjoys the non transferable part $h$.

The authority is able to internalize the benefits of the investment on the rolling stock even if in an indirect manner, acquiring the incumbent assets. Hence, in order to induce more investment in the first period, the selection is biased in favour of the incumbent, the larger the transferable effect on the rolling stock (Figure 8.A.).

![Figure 8.A.](image)

Figure 8.A.: probability for $I$ to be active in $t=2$, according to $\beta^I$ for different values of effect of transferable investment ($g$) on $\Delta_{PUB}$ according to $\beta^I$

Moreover, the larger the transferable part the lower the probability to have the competitor favoured (Figure 8.B.).
Figure 8.B.: effect of transferable part of the investment \((g)\) on \(\Delta_{PUB}\) according to \(\beta^i\)

**Remark 3.B:** if the first period investment has a high transferable effect on the rolling stock, then the incumbent has higher probability to remain active. Asymmetric information favours the competitor less, the larger the transferable effect.

Consider now the non transferable part of the investment.

If the share of non-transferable effect increases, the incumbent enjoys again an advantage (Figure 9.A.), because the lower is the breakout rule chosen by the authority, that is the incumbent has a higher probability to be active in the second period.

Contrary to what found for the transferable part of the investment, once the competitor is efficient enough, the probability of having the competitor favoured increases with the share of non-transferable effect.

The authority does not need to induce more investment because only the incumbent benefits of it and already has an incentive in investing.
Figure 9.A.: probability for I to be active in t=2, according to $\beta^I$ for different values of the transferable part of the investment ($h$)

Figure 9.B.: effect of non transferable part of the investment ($h$) on $\Delta_{PUB}$ according to $\beta^I$

Remark 4.B: the greater the non-transferable part of the effect on the rolling stock, the larger the probability for the incumbent to keep on providing the service and the larger the probability of having the competitor favoured.
**Effect on the second period cost**

If the incumbent invests in the first period, then he can exploit a reduction in second period costs. The higher is this effect, the lower is the threshold chosen by the authority: if the investment has a high impact on the second period costs, then the authority is willing to accept only very efficient competitor (Figure 10.A.). If the competitor has a cost parameter equal to the threshold, then she is more favoured, the larger the effect on second period costs (Figure 10.B.).

![Effect of the Reduction in Costs](image)

**Figure 10.A.:** probability for I to be active in t=2, according to $\beta'$ for different values of the reduction in second period costs due to first period investment ($k$)
Effect of the Reduction in Costs

Figure 10.B.: effect of the reduction in second period costs due to first period investment (k) on $\Delta_{PUB}$, according to $\beta^I$

Remark 5.B: if the reduction in second period cost is high, the authority chooses a low threshold. If the cost parameter of C equals the threshold, the larger the effect on the second period cost, the higher the probability to have the competitor favoured.

Effect of the reward

In the specification of the model we considered the reward for the incumbent in the first period and for the second period provider to be a share ($\theta$) of the value that the firm would assign to the rolling stock in case of private ownership. Under asymmetric information, the reward that the authority grants has an effect on the rent, so that the higher the reward, the higher the rent, the lower the investment required. The authority incentivizes the investment by increasing the probability of being active in the second period (Figure 11.A.).
Figure 11.A.: probability for I to be active in t=2, according to $\beta^i$ for different values of the reward ($\theta$)

Asymmetric information favours the competitor more, the lower the reward (Figure 11.B.).

Figure 11.B.: effect of the reward ($\theta$) on $\Delta_{PUR}$, according to $\beta^i$
**Remark 6.B:** the larger the reward the incumbent receives, the larger the probability to select the incumbent. Asymmetric information favours the competitor for low reward.

**Effect of the impact on social utility**

The public ownership in the second period positively affects the social utility and the effect depends on the investment undertaken in that period. In our specification the parameter that represents this increase as a part of the effect on the rolling stock is \( \omega \).

If the impact is *per se* already strong, i.e. even in case of a small investment the society gains a lot, then the authority prefers to bias the selection in favour of the competitor in order to exploit the advantage from a lower rent to the incumbent in the first period (Figure 12.A.):

![Effect of the Impact on Social Utility](image)

**Figure 12.A.:** probability for I to be active in t=2, according to \( \beta' \) for different values of the impact on the social utility (\( \omega \))

The larger is the effect on social utility, the more the competitor is favoured (Figure 12.B.).
Figure 12.B.: effect of the impact on the social utility (ω) on Δ_{PUB}, according to β^I

**Remark 7.B:** the larger the impact on the social utility, the lower the probability to be active in the second period for the incumbent and the larger the probability to have the competitor favoured.

To summarize, under public ownership the larger the effect of the investment on the costs or on the rolling stock, the larger is the probability for the incumbent to remain active. As highlighted in the private ownership context, the first period activity of the monopolist gives him an advantage over the competitor.

Only the increase in the cost of public funds or of the impact on social utility lowers the probability for the incumbent to be chosen.

Finally, notice that the asymmetric information over the cost parameter of the firm tends sometimes to favour the competitor and sometimes to favour the incumbent.
8. Concluding remarks and possible extensions

Having the aim of a potentially more competitive environment for the local rail market, the problem of the rolling stock supply has to be tackled. The assessment of the solutions proposed certainly has to take into account the incentive to invest, because maintenance and investment in security represent consistent financial investments for the firm but also does the investment in the quality of the service.

As seen in the model above, the optimal levels of investment is sensible to the choice of the ownership of the rolling stock in the second period as in the two frameworks the firms and the authority are able to internalize the effect on costs and rolling stock different ways.

In the case of complete information, the cost of public funds and the benefit the society obtains from having the local authority as the owner of the assets play a crucial role in defining in which context there is a higher level of investment.

If the cost of public funds are low enough and the marginal increase in the additional utility is high, then the public ownership leads to larger amount of investment.

Unfortunately, the local authority does not have all the information about the costs of the firm.

Under asymmetric information, there is a trade-off between the effect of the investment and the rent from asymmetric information and it is not possible to obtain clear-cut results.

As this model refers to the local markets, it leads us to conclude that a one-fit-all solution cannot be applied. If the aim is to increase the investment of the provider to supply a better service in terms of quality (i.e. well maintained rolling stock, cleaning service, etc), it is necessary to evaluate the market and decide how to solve rolling stock issue in that particular market, according to its characteristics.

It is necessary to consider carefully the cost of public funds, because a high cost reduces the optimal level of investment and the effect is probably stronger under public ownership, because the local authority acquires the rolling stock and also pays a reward for the investment undertaken by the provider.

Therefore, the local authority has to decide if the benefits generated by the investment are able to overcome the cost of buying the assets. In
our model we assumed that the public ownership of the assets generates an additional social utility: if it is large enough, then buying the rolling stock would be an appropriate answer to the need of increasing investment.

Studying the probability for the incumbent to be selected in the second period – even if for a particular specification of the model – we found that the first period investment increases the probability to be active in the second period for the incumbent and this is true both under private and under public ownership. This finding is in line with what we observe at least in Italy: the company operating the line is very often the same that will be active for the next period contract.

Only the increase in the cost of public funds lowers the probability for the incumbent to be selected.

In this paper we did not consider the level of subsidies but it would be interesting to check which framework calls for lower transfers (or at least for which characteristics of the market) and study the impact of each arrangement on total welfare.

In the model, the game is designed such that the authority announces at the beginning of the first period in which framework the provider operates. But consider the case in which even if the incumbent knows that in the next period the supplier will be selected, the authority does not reveal its choice about the type of ownership. What we expect is to have the incumbent investing less in the first period. When the incumbent invests in the first period, he is no longer uncertain only over the result of the selection but also over the probability of being active in the second period as the rule of the selection is not known.

Finally, we considered only one type of investment that positively affects both the costs and the value of rolling stock. What else if the incumbent could undertake an investment which lowers cost but also quality (as Hart 2003 or as the negative externality in Bennett and Iossa)? If the authority announces that it will buy the assets, then it might be the case that the incumbent lowers the quality of the rolling stock, in case he has low probability to be active in the second period.
Chapter 3

Universal and Public Service Obligations

1. Introduction

The concepts of universal service (US) and of public service obligations (PSO) refer to general economic interest service that is services that public authorities consider of general interest but which would not be provided if left to the commercial interest of firms.

The liberalization of the railway sector requires a particular attention to the issue of universal and public service obligation. The entry of new firms is likely to happen in profitable segments. Therefore, the previous monopolist is probably no longer able to cross subsidize between profitable and unprofitable provisions of services.

Once a Member State recognizes the general interest connected to an activity, it can implement a special discipline in order to achieve its objective that is the provision of the service under specific conditions.

A possible device is to provide for public service obligation. There is not a unique definition of this concept, but according to some authors it is a specific provision or a specific way of providing an activity (in terms of regularity, continuity, quality, security, etc.) in line with what public authority recognizes to be of public interest.

The universal service is a way of implementing this obligation. The European Commission (2001) defines the universal service obligation as the obligation to provide a specific service on the entire territory at accessible prices and at similar quality, regardless of the profitability of the operation.

The universal service obligation is referred to in the European legislation only in the telecommunications and postal sector.

Each Member State evaluates the need for service obligations according to its domestic conditions and therefore there is not a unique definition and implementation of USO. For example, in Italy for both the telecommunication and the postal sector the law implementing EU

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17 See Sorace, 1999
directives defines in details the services subject to universal service obligations and provide for the set up of a fund to finance US where the operators contribute according to certain rule\textsuperscript{18}.

The railway sector is not subject to a discipline of universal service obligations, even if according to some authors (Cartei, 2002) the reference to “sufficient services”, “particular tariff conditions” and to “specific passengers’ categories”, could represent the key elements to define a universal service discipline also in the railway sector.

As for the transport sector, the European and the national legislation only refer to public service obligation that is “obligations which the transport undertaking in question, if it were considering its own commercial interests, would not assume or would not assume to the same extent or under the same conditions” (Reg. 1191/69). There are three components to consider: the obligation to operate, the obligation to carry and tariff obligations.

The obligation to operate mainly refers to any obligation imposed on an operator to ensure the provision of a transport service satisfying fixed standard of continuity, regularity and capacity.

The obligation to carry means that the undertaking is obliged to accept and carry passengers at specific rates and conditions.

Finally, the tariff obligation requires the undertakings to apply special tariffs to certain categories of passengers or on certain routes.

It is interesting to note that neither the USO nor the PSO are static concepts as the European Commissions establishes that in the telecoms and in the postal sector the set of services has to be defined according to technological process, market evolution and consumers demand. Also for the railway sector PSOs can be revised as obligations can be ceased if a different intermodal service is active.

Therefore, the evaluation of which routes are subject to public service obligations is necessarily forward looking, that is public authority takes into account which routes could be abandoned in the absence of adequate incentives and the existence of alternative mode of transport.

The demand side analysis is fundamental: the liberalization of a sector in which the demand is slowly increasing or is declining leads to the

\textsuperscript{18} For the telecommunication sector see Laffont and Tirole 2000 and Cambini, 2002. For the postal sector see Visco, 2003
serious threat for the incumbent subject to service obligations of loosing revenues with the opening up of the market. As underlined at the beginning of this section, new entrants will compete for the profitable market, reducing the incumbent’s revenues and possibly increasing his deficit due to PSOs.

Moreover, the definition of the obligations has to look at the economies of the service. In railways, we distinguish between economies of densities and economies of size (Seabright et al., 2003). The first refer to the change in costs due to the increase in traffic, given the network. The latter represents the change in costs given the density, but varying the network. Empirical analyses highlight the existence of economies of density, as if traffic increases on a certain network, the passenger – kilometre cost decreases, while the results for economies of size are not clear cut. For example, Cantos (2001) find that returns to density for the main European operators vary from 1.42 to 2.04, while returns to scale vary between 0.45 to 1.4.

If the sector is characterized by technological progress, then it is more likely to observe the raise of new profitable niches or segments in the market. This aspect could reduce the impact of service obligations on the costs. The railway industry does not show technological advances as in telecommunications, but is not as static as the postal sector. Consider for example, the high speed transport, whose development calls for new revenues.

The use of universal service concept only in telecoms and postal service justifies the extensive literature applied to these two sectors.

Studies on railways are still limited and the existing ones are mainly reports about the solutions adopted by different countries as for example the information gathered in the Liberalization Index Studies (2002, 2004, and 2007) and in the CER\textsuperscript{19} report (2005).

In the next sections we review some of the main studies about universal service obligation.

2. Rationale for USO and costs

The rationale for the existence of the USO is often attributed (Cremer et al., 1998 and 2001) to the existence of network externalities, to a

\textsuperscript{19} Community of European Railway and Infrastructure Companies
redistributive objective of the public authorities and to the public good characteristics of some services.

If an activity has network externality, the promotion of its diffusion avoid that the participation is too low, depressing the development of the network. But network externalities do not completely support the imposition of USO because not in all sectors characterized by network externalities it is possible to justify the implementation of USO and because some of the USO constraints do not appear to have the promotion of network externalities as object.

USO can also be interpreted as a redistributive device towards high cost consumers when prices only partially represent cost differentials and towards low-income individuals if special tariffs are designed for specific social categories.

Finally, the service at glance often has the characteristics of public good, so that from a social point of view its existence is worthy per se.

It is also important to notice that universal service obligation (USO) can be an instrument of regional policy, especially if the spread of services helps in sustaining the development of the region or prevent from the decline particular areas.

Extensive attention was devoted to the issue of how to measure the cost of USO.

One possible way is to consider the profitability cost that is defined as the loss in profits incurred by the operator due to the USO, as the Net Avoidable Cost. On the other hand, one can consider the welfare cost, defined as the deadweight loss implied by USO.

Panzar (2000) defines the method to measure the cost of fulfilling a universal service obligation, based on the computation of additional resource costs incurred and foregone revenues.

The process starts with the definition of the set of services included in USO, bearing in mind some characteristics, such as the strength of network effects, the costs and the redistributive goals. Customers have to be divided into two groups according to the fact that they would or would not buy without service obligations.

To understand the cost of PSO it is necessary to consider an hypothetical competitive market alternative and check the incremental
costs due to service obligations and the foregone revenues, which depend on the subsidy mechanism.

Even if from a logical point of view this is a correct method, it requires a lot of information which are rarely available, such as a specific information about customers (for example, would customer x buy without USO or not?).

Therefore, Panzar suggests a more direct way of determining the cost of USO, equal to the losses due to the connection of subscribers for whom the incremental cost of connection exceeds the socially determined maximum allowable rate.

As for the costs, Panzar proposes to rely on the costs that would actually be incurred by the provision of additional service relative to the unsubsidized market benchmark, to define the floor for the subsidies, while the ceiling on subsidy is measured by the stand-alone cost of providing the services.

Rodriguez and Storer (2000) suggest using the Entry Pricing approach. They analyse the postal sector and compare the Net Avoided Cost (NAC) approach with the Entry Pricing (EP) approach. The advantage of the latter one accounts not only for the avoidable costs but also for the effect of changing in the market structure.

Recall that in the postal sector there exists an area of reserved activities that cross - subsidize USOs. The constraint is represented by uniform prices and geographical constraint so that all consumers must be connected to the network.

The authors argue that if the market structure is stable, then the NAC is an appropriate measure, even if when – as in the postal sector - there exists a reservation area it is of little practical use.

The EP approach appears to be more appropriate if the sector is subject to ongoing modifications, due for example to liberalization as competition and entry could occur, because while NAC assumes a value, EP cost varies according to market condition. The larger the number of competitors, the lower the profits for the US provider on profitable routes. Therefore, the use of NAC to evaluate the cost of universal service if entry occurs could underestimate the real cost of providing the service at the same uniform price pre entry.
3. Allocation and funding of service obligations

In this section we review the main insights of the literature about the allocation and financing of universal service obligations.

We are aware that the concept of universal service applies only to the telecommunication and postal sector, but as there are strong similarities with public service obligations, this literature represents the appropriate reference also for the railway sector.

Recall that the increasing attention to the issue of service obligations is linked to the opening of sectors previously served by a monopolist able to cross subsidize between profitable and non profitable markets. As soon as the market is open to new firms, concerns about entry and financing arise. As for entry, one can expect “too much” entry in the profitable segments and “too little” in the unprofitable one, unless there is some mechanism inducing firms to compete for the segment subject to service obligations (Armstrong, 2001). Moreover, there is also a difficulty in financing if entry occurs only in the profitable market as this would limit the ability of the incumbent to cross subsidize across markets.

Therefore, at least two main areas can be analyzed: how to allocate service obligations among providers and how to finance the deficit.

As for the first issue, it could be the case that there is a single designated operator that provides the service or that the operator is selected through an auction.

In most of the countries, the firm providing the USO before liberalization keeps on providing it, but it can also happen that the authority responsible for service obligation chooses a different firm. We refer to this framework as restricted entry regulation that is the competitor is not allowed to serve USO area.

However, a franchising system is a feasible alternative and it would have the advantage of potentially assign the USO to the most efficient provider.

In between restricted entry and franchising, extensive attention has been devoted to pay or play (PoP) system, in which the entrant can decide not to pay the fee or tax required to finance the USO and serve non profitable consumers.
As for the financing issue, the mechanism could rely on funds raised by taxpayers either connected to the activity or not.

If the mechanism is self funded, the deficit is covered by cross subsidies or taxes levied on consumers or firms involved in the markets. For example, the regulator can set up a fund where operators of the sector participate paying a fix fee or a specific tax levied on sales or through access surcharges.

The other possibility is to finance the transfers through taxes levied on taxpayers, not necessarily participating into the market. This is the case for general taxation or for a fund common to the entire sector. Consider railways: not only railway operators could be require to contribute to the fund, but all transport operators, regardless of the mode of transport, contribute on the basis of a defined criterion as for example a measure of the negative environmental externalities they produce.

A comparison of different financing mechanisms has been proposed by Gasmi et al. (2000). They use a forward looking engineering model to simulate possible market equilibria and welfare under different entry scenarios and regulatory intervention in the telecommunication sector. They show that for value of the cost of public funds above a certain threshold the use of cross subsidy still grant a higher welfare then when universal service is funded through cross subsidy. This is an interesting result, especially for developing countries, which often satisfy these thresholds.

Service obligations are a constraint on the firm activity and have an impact on all connected markets.

The relation between the markets is almost always modelled by the imposition of a uniform price for all the consumers regardless of the profitability of the segment. This kind of constraint is sometimes referred to as non discrimination constraint: the same tariff should be proposed to all consumers, whatever their location or their connection cost.

But according to the definition of USO, also a ubiquity constraint can be imposed. The ubiquity constraint states that all consumers should be connected to a network, whatever their location and can be imposed together or independently of the non discrimination constraint.
Moreover, it could also be the case that special low tariffs that can be imposed for specific consumers, as for example for the customers in the rural area in telecommunications.

In what follows we illustrate the main results of the literature and consider first the allocation and financing of service obligations for a given quality and size of the area subject to constraints and then consider the case for firms deciding the extension of obligations and quality of the service.

In order to make the comparison of results easier, we refer to a general unified framework. Consider a market $M$ characterized by heterogeneous consumers, which can be grouped in two different classes on the basis of their connection costs: the low cost group $\mu$ and the high cost group $\bar{\mu}$, whose proportion in the population is respectively $\alpha$ and $\bar{\alpha}$.

Market $M$ is divided into two segments: $\overline{M}$ where the service obligation is imposed because of the presence of group $\overline{\mu}$ and $M$ where all consumers are of type $\mu$ and firms are not constrained by USO.

### 3.1. Allocation and financing with fixed quality and USO extension

The most frequently case analyzed in the literature involves a regulator deciding how to allocate and finance service obligation for a service of a given quality on a certain area, i.e. to a defined group of consumers.

A general finding is that when a firm operates both in a market with service obligations and in a competitive market, which are strategically linked, then it is less aggressive in the latter one.

If the obligation is imposed on a certain operator, than the welfare effects depend both on the interplay between allocation mechanism and financing.

If the USO provider is chosen on the basis of an auction, then it is usually possible to obtain an increase in welfare under not too restrictive conditions.
Hoering et al (2002) study the relation between regulation and competition if the aim is to promote competition and implement universal service.

The existence of USO induces some distortions when competition takes place. First of all, inefficient entry may occur and in this case, they show that the set up of universal service fund can align private with social incentives. Secondly, the entry of a new provider reduces the profits in profitable market and this calls for a higher level of subsidy. Finally, if the link between markets is given by uniform pricing, then firms’ competition is less aggressive, so that equilibrium prices are higher and the deadweight loss increases.

Chonè et al (2000) analyze the welfare effect of restricted entry (RE) regulation and pay or play rule (PoP), when service obligations are characterized by ubiquity (U) and/or spatial non discrimination constraints (UND/ND).

With restricted entry regulation, the USOs are financed through taxes t levied on the profitable unit sold, so that for the incumbent it is an internal transfer and creates no distortion, while if the competitor serves the profitable market, then taxes are levied on his units sold and transfers to the incumbent take place.

Under pay or play regulation, the competitor can serve non profitable consumers, but does not pay nor receive taxes, so that PSOs are funded by cross subsidies - either by the incumbent or by the competitor - or by a taxation as described for the restricted entry regulation.

Chonè et al. (2002) extend the analysis considering also the possibility for the regulator to use transfers T.

The results about which funding system leads to best results in terms of welfare $W$, depends both on the regulatory mechanism and on the constraints imposed.

From consumers points of view, if service obligations require only the ubiquity constraint, high cost consumers prefer the PoP regulation both when the incumbent cross subsidizes between the two markets and when the taxation regime is implemented. But if the competitor cross subsidizes both segments, then the result is less clear cut because there is a multiplicity of equilibria, allocating differently the share of surplus between consumers and the firm.
As for total welfare, when only ubiquity constraint is imposed, they show that it is at least higher under pay or play regulation than under restricted entry regulation. The result is due to the possibility that under PoP, both markets are served by an entrant more efficient than the incumbent.

Consider now the case of service obligations characterized both by the ubiquity and the non discrimination constraint: the result about welfare comparison no longer holds.

When USO is characterized by the ubiquity constraint, then at equilibrium $W^{PoP} \geq W^{RE}$. If USO also satisfies the non discrimination constraint, then the same result holds if the tax rate is lower than a threshold $\tilde{t}$.

By contrast to the case in which only ubiquity constraint is imposed, if also non discrimination has to be fulfilled, then the welfare under PoP can be strictly lower than under restricted entry regulation if the tax rate is above a certain threshold.

The analysis thus highlights that the goodness of a certain funding or allocation mechanism cannot be evaluated per se, but it is necessary to consider the implementation conditions and the specific constraint imposed for USOs.

Mirabel et al. (2008) focus on the compensation scheme in the funding mechanism. They show that a mix of unit and lump sum subsidy can help to reduce the distortions induced by USO.

They also extend the analysis to the case of endogenous choice of the universal service provider. The subsidy mix solution appears to be welfare equivalent to play or pay regulation and superior to franchise bidding.

The allocation of service obligation can take place also through auctions. Sorana (2000) compares the use of the Carrier of Last Resort (COLR) auctions with a traditional uniform subsidy scheme (EPOS) paying the same subsidy for any consumer served in the area under consideration at no more than the regulated price. The objective of these auctions is the provision of specified services at a given maximum price. Consider only the high cost group $\bar{\mu}$, which is now composed of $M$ types, ranked on the basis of their connection costs. Therefore, $\bar{\mu}_1 < ... < \bar{\mu}_h < ... < \bar{\mu}_{M - 1} < \bar{\mu}_M$, with $h = 1,...,M$. Moreover,
we consider the case of firms with approximately the same cost structure.

The analysis shows that the COLR auctions grant a reduction in the level of subsidy required. The intuition is related to the impossibility for the regulator to use its knowledge about the different costs of serving the consumers under EPOS: the uniform subsidy must cover the cost of the highest costly customer, while with the auction bidders average through consumers’ costs.

Consider firms with similar cost structure. If there exists at least a type of consumer that cannot be served for less than the second lowest average cost, a second price COLR auction asks for lower subsidy $T$, than the uniform subsidy scheme: $T_{COLR} \leq T_{EPOS}$. The result holds both when subcontracting is allowed and when it is not.

In general, the previous result holds without subcontracting even if firms do not have the same cost structure. With subcontracting, COLR is cheaper than EPOS also under other conditions as a sufficient heterogeneity of consumers with respect to firms costs or if the winner of the auction can obtain a sufficiently large share of the gain from subcontracting.

As for total welfare, a sufficient condition for $W_{COLR} > W_{EPOS}$ is that the cost of public funds is higher than the weights assigned to both consumers surplus and firms profits.

The issue of the allocation of USO through auction is also addressed by Anton et al. (2000, 2002), but they consider a different constraint: firms bid for the subsidy needed to fulfil USO over an area and the price of the service is determined by competition in another market. More precisely, the winner of the auction has the obligation to provide the service at the same price prevailing in the profitable segment, where competition in quantities between firms takes place.

The firm serving both markets $M$ and $\bar{M}$ becomes a softer competitor, so that the equilibrium prices are higher than under unconstrained competition in quantities. More precisely, the equilibrium is not symmetric and the firm providing service in both markets obtains a lower profit than when it operates only in $\bar{M}$.

Thus the equilibrium subsidy compensate for the strategic disadvantage.
The presence of a firm in both markets, put her at a strategic disadvantage and therefore the equilibrium subsidy contains a premium to compensate for the strategic disadvantage.

Total welfare increases when the auction takes place and price constraint is imposed, if the regulator gives to profitable and non-profitable market similar weights.

The analysis is extended to the case of outside firms, i.e. firms not operating in the profitable market, allowed to bid for the rural market. This extension is interesting because the outside firm cannot affect the price in the profitable market, so that if it wins the auction, there is no longer a strategic link between markets. Therefore, the negative effect on consumers’ welfare can be neutralized if the outside firm wins the auction.

3.2. Allocation and financing of USO: the role of quality and coverage

In the previous subsection, we considered the case of USO when the quality $\theta$ and the coverage $x$ is fixed. Now we check for main results when the two characteristics $\theta$ and $x$ are to be determined.

According to Valletti et al. (2002) and Hoering et al. (2002) if a coverage constraint is imposed together with uniform pricing, then equilibrium prices and coverages are higher.

If we analyze the welfare components, it appears that firms profits have an opposite path: the incumbent profits decrease, while the entrant profits increase. The consumers surplus increases only if fixed costs connected to the activity in an area do not increase too fast.

Suppose that each firm can choose the level of coverage. With respect to the benchmark case in which no USO is imposed, if the incumbent $I$ is subject to uniform price $UP$, he chooses a lower coverage in order to weaken competition $x_{I}^{UP} < x_{I}^{noUSO}$. The entrant $E$ may also choose to cover a less extended area.

The same result holds if not too tight price cap is imposed.

Calzada (2009) extends the study of the coverage to the relation with quality $\theta$. 
If the incumbent can choose both coverage and level of quality, then he would choose a larger area than when quality is fixed: $x(\theta)_i^{UP} > x_i^{UP}$.

On the other hand, the entrant can strategically choose either the extension of the coverage or the product differentiation to weaken competition.

In order to countervail the strategic effect, the regulator could impose minimum coverage or quality standards.


Instead of defining the area subject to USO according to the connection cost, the regulator chooses to identify it on the basis of total welfare. This choice allows reaching the second best allocation if the unit subsidy instrument is available.

Sorana (2000) analyses COLR auctions organized to compete in the market, so that the quality of the service plays a role. The main issue in this case is that auction is vulnerable to collusion.

In a per-subscriber subsidy action, bidders are more likely to collude, than in standard procurement auction or COLR auction for lump sum subsidy.

The intuition for this result is that as there exist more than one provider of the service, in case of defection the punishment occurs immediately and firms do not have to wait next auction.

Sorana shows that a device for reducing concerning on collusion is to let the number of providers be endogenously determined.

4. Concluding remarks

In this chapter we briefly assess some of the main insights of the literature about universal service obligations.

As underlined at the beginning, in the European Union the concept only refers to telecommunication and postal sector.

As for the railway industry, the European directives refer to public service obligation. Even if from a normative point of view they are two different concepts, the economic analysis can be performed almost in the same way.

We focused on two issues, the allocation and the funding of service obligations both when the quality is given and when it is not.
The main result is that the existence of USO creates distortions and that a firm operating also in a competitive market tends to be less aggressive.
Chapter 4

Public Service Obligations and Unknown Demand

1. Introduction

The aim of this paper is to study the effect of public service obligations on total welfare according to different kind of regulation.

The reform of the railway industry has to account for the possibility that the liberalization induces railway operator to dismiss some unprofitable lines. Therefore, it could be necessary to intervene at State level in order to ensure that socially valuable routes keep on being supplied. In this case, each Member State can implement Public Service Obligations (PSO).

“Public service obligations means obligations which the transport undertaking in question, if it were considering its own commercial interests, would not assume or would not assume to the same extent or under the same conditions”\textsuperscript{20}.

The main questions about PSOs are relative to how to allocate these obligations and how to finance them.

As for the first issue, the obligation can be imposed on an operator designated by the public authority (restricted entry regulation) or a franchise bidding can take place. An alternative is the pay or play mechanism, in which the entrant can decide either to pay a tax to finance USO or to serve the unprofitable segment.

As for the financing issue, the deficit is usually covered by cross subsidies or by taxes on consumers or operator participating to the market.

Chonè et al. (2000 and 2002) analyze the allocation of the obligation through pay or play and restricted entry regulation considering different kind of universal service constraint. The allocation of service obligation through actions is studied by Sorana (2000) and Anton et al. (2000 and 2002). The main result of these papers is that the existence of

\textsuperscript{20} Reg. EC 1191/69, art. 2
service obligations leads to distortion in the other markets, where the US provider operates. Moreover, the appropriate choice of financing mechanism can improve welfare in all allocation mechanism.

In the past, the incumbent firm was a regulated monopolist that provided all kind of services, receiving transfers in return. Therefore, we start analyzing this framework.

But the opening up of the market requires a new organization of the sector. In particular, we consider first the case of a Transportation Authority in charge of defining only the transfers that ensure non negative profit to the incumbent when he is constrained by PSOs and has to compete with another firm.

Finally, we introduce a form of participation by all firms in the industry to cover the deficit generated by PSOs: each of them pays a share of the profits on the competitive segment into a common fund.

This kind of solution to the problem of PSO financing has already been adopted both in the telecommunications and in the postal sector.

Moreover, we add an asymmetric information issue. A feature of the railway sector is the difficulties encountered in gathering disaggregate data which are necessary to establish the exact financial need of routes under PSO. Collecting data over passengers travelling on a certain route by somebody else than the provider of the service is very costly and seems to be almost impossible. Even if railways operators or association of operators or transport authorities publish data, such as passenger kilometres, train kilometres or number of passengers, at the aggregate level, it appears that nobody else but the operator of a line knows how many passengers have travelled on that route.

Therefore, in the second part of the paper we introduce asymmetric information over demand in one of the market and study how welfare is affected.

The literature about asymmetric information is really vast (Laffont and Tirole, 1993, Laffont and Martimort, 2002; Armstrong and Vickers, 2000; Armstrong 1998) but not many papers relate the question of asymmetric information over demand and optimal market structure (Iossa, 1999).

Moreover, the concept, the aspects and the constraints imposed by universal service in telecommunications and in the postal sector has
been deeply analyzed. A characteristic of these papers is that the link between markets is almost always due to a uniform price constraint.

In this paper we consider a different approach, instead: a price constraint is imposed only on one market, but the existence of imperfect substitutes for the service under PSO, makes this constraint affecting also the outcome of the other markets.

It appears that the PSO induces distortions also in the other markets due to the substitutability among services. Moreover, using some simulations we show that the implementation of the PSO fund grants an increase in total welfare, relative to the multiproduct regulated monopolist and to the case of simple introduction of competition.

In the next section we present the model and analyze what happens under complete information when first there is a multiproduct regulated firm, then when there is competition in one market and finally when the PSO fund is set up. Section 3 proposes a comparison of the equilibrium outcomes. Section 4 introduces and solves the problem of asymmetric information over demand. Section 5 illustrates the effects on firms’ choice and on welfare. Section 6 concludes.

2. The model

A railway network connecting the same origin-destination is composed of three different lines.

Consider for example three feasible choices to go from Milan to Rome: passengers can travel passing through Genoa with a Western line or they can almost vertically cross the country using a North-to-South connection or finally they get to Rome through Bologna on the Eastern route. The three services are imperfect substitutes because of, for example different travel time. In general, passenger transportation involves both complementarity and substitutability among services, but in this paper we want to focus on their substitutability.

As explained in the introduction, the PSO in the railway industry is an existing constraint on the activity of the operators. Suppose that the provider of the Western line (service 1) has to fulfil PSO: the need for this obligation is determined according to the evaluation of the positive social surplus that it produces. Therefore, the regulator defines a price for this service and grants transfers to the operator. By definition, the price for this service \( p_1 \) is smaller than its marginal cost \( c_1 \), \( p_1 < c_1 \).
However, we do not investigate how this price is decided upon and take it as exogenous. This is not too far from the situation observed in real life as the price seems to be decided according both to political/social objectives and to economic reasoning, where the first objectives are weighted more than the second. Moreover, prices are usually defined on the basis of the operated train kilometres, but the number of tickets sold is known only ex post and it is obviously an important variable for train operators.

The authority in charge of establishing the price for public service obligation directly negotiates with the incumbent on prices and we consider it fixed at level $p_1$.

The constraint on this market and its management operated by the incumbent firm $I$ is unchanged throughout the paper.

We will define this price as the percentage of the constant marginal cost $c_1$ that the authority wants to be covered by prices, i.e. establishing $p_1$ the authority decides how much it is willing to subsidize each unit of output.

The other two services – in our previous example North-to-South and Eastern line - are provided either under price regulation by a single monopolist or under competition over prices. We denote these services as service 2 and 3.

We consider different market structures and analyze the equilibrium outcome in terms of prices and transfers. In particular, assuming service 1 under PSO, with $p_1 < c_1$ and operated by the incumbent, we analyze first the case in which both services 2 and 3 are provided by the regulated incumbent $I$, who receives transfers in return.

Then we study what happens if competition between services 2 and 3 takes place, so that they are respectively provided by the incumbent $I$ and by a new firm $E$.

In a first scenario, firms compete in prices and the regulator is in charge of defining the price for PSO and the transfers.

In a second competitive scenario a PSO fund is set up. Each provider operating in one of the competitive segments has to contribute with a share of his net profits. In this case, the regulator defines the percentage of contribution to the fund $k$ and the transfers.
The example above is only a way of explaining how our framework applies to the railway industry. But it could also be interpreted as different kind of services operating on the same line and with different qualities, for example high speed train v. traditional train. In what follows we stick to our first example as we do not introduce the evaluation of quality.

In order to understand the effects of the different market structures, we specify the demand functions with some symmetry between products:

Market 1, PSO market: $q_1 = a_1 - b_1 p_1 + g (p_2 + p_3)$  
(because $b_i > h > g$)

Market 2: $q_2 = a_2 \cdot \theta - b_2 p_2 + g p_1 + h p_3$

Market 3: $q_3 = a_3 - b_3 p_3 + g p_1 + h p_2$

In the second part of the paper, we analyze how asymmetric information over demand (more precisely, over $\theta$) in market 2 can affect the previous results.

2.1. Multiproduct regulated monopolist

Consider first the case that more closely represents what the organization of the sector has traditionally been: the incumbent provided the transportation service as a monopolist on the entire network on the basis of some kind of regulation.

The operator $I$ is in charge of supplying service 1 according to PSO, so that the price in this market is fixed at $p_1$ and he receives transfers. Note (see the above demand specifications) that even if the price in this market is fixed, the quantity is not, as it depends on the prices of the other two markets: the increase in the price of one of the other markets, positively affects the quantity of the PSO service.

Firm $I$ operates in all markets as a monopolist, but the prices for market 2 and 3 are regulated by the Transportation Authority.

The cost function to provide service $i$ is $C(q_i) = c_i q_i$  
(i=1, 2, 3).

Define $S(q_i)$ the surplus that the consumers obtain using service $i$ and $\lambda \geq 0$ the cost of public funds.
The utility function of the multiproduct regulated monopolist $I$ is the sum of the utility it obtains from each service and of the transfers it receives:

$$U^I = U_1 + U_2 + U_3 + T$$  \hspace{1cm} (1.)

or

$$U^I = p_1q_1 - C_1(q_1) + p_2q_2 - C_2(q_2) + p_3q_3 - C_3(q_3) + T$$

The regulator maximises the welfare function $W$ given by the sum of net consumer surplus, utility of the firm and transfers paid by the regulator

$$W = \sum_{i=1}^{3} \left[ S(q_i) - p_iq_i \right] + U^I - (1 + \lambda)T.$$  \hspace{1cm} (2.)

The objective function can be rewritten as:

$$W = \sum_{i=1}^{3} \left[ S(q_i) - (1 + \lambda)C_i(q_i) + \lambda p_iq_i \right] - \lambda U^I.$$  \hspace{1cm} (2.)

Under complete information, the regulator can maximize the welfare function, setting $U_I = 0$.

**Proposition 1:**

In case of a multiproduct regulated monopolist, when there exists a PSO on market 1, the prices on market 2 and 3 are such that:

$$\frac{p_i - c_i}{p_i} = \frac{\lambda}{1 + \lambda} \frac{1}{\hat{\epsilon}_i} \left( \frac{\epsilon_{i,j}}{1 + \epsilon_{i,j}} \right) \left( \frac{p_1 - c_1}{q_1} \right)$$  \hspace{1cm} (3.)

in which $\epsilon_{ij} = \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i} > 0$, $\epsilon_i = -\frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i} > 0$ and $\epsilon_i > \epsilon_{ij}$

$$\hat{\epsilon}_i = \epsilon_i \frac{1 - \frac{\epsilon_{j,i}}{\epsilon_j}}{1 + \frac{\epsilon_{j,i}}{\epsilon_j} \frac{q_j p_j}{\epsilon_j q_i p_i}}$$  \hspace{1cm} \text{and}  \hspace{1cm} \hat{\epsilon}_1 = \epsilon_i \frac{1 - \frac{\epsilon_{j,i}}{\epsilon_j}}{1 + \frac{\epsilon_{j,i}}{\epsilon_i} \frac{\epsilon_{j,i}}{\epsilon_j}}$$

with $i, j = 2, 3$.

The transfers $T$ are such that $U^I = U_1 + U_2 + U_3 + T = 0$.  \hspace{1cm} (4.)
As $p_1 - c_1 < 0$, the second part on the right hand side of (3.) is negative. The obligation to serve market 1 at a price lower than the marginal cost creates a distortion not only for this service, but also for all other services, as they are imperfect substitutes.

If the Transportation Authority is willing to heavily subsidize public service obligations, than she also affects the outcome of the other two markets, decreasing their optimal prices.

Obviously, the lower the unit subsidy $(p_1 - c_1)$, i.e. the closer the price to the marginal cost, the lower the distortion induced on the other markets.

Notice also that the distortion of the price $i$ depends on the ratio between the deficit due to PSO and the revenues from market $i$, so that other thing equal the more profitable the market, the smaller is the distortion.

2.2. Multiproduct firm and single product competitor

In this section we consider the effect of the introduction of competition. One of the principles of the European reform is the opening of the sector to all the firms able to provide transportation service. Each country can decide to keep some area of the activity under PSO, on the basis of the objectives we recall in the introduction. So it is interesting to understand the effect of competition when one of the markets is constrained by PSO.

In this framework, we assume that the regulator cannot implement any regulation except the definition of $p_1$ and the transfer. Notice that in this framework the regulator does not have a mechanism that induces the revelation of the true $\theta$ under asymmetric information over this parameter. She simply has the role of paying for the incumbent deficit.

The incumbent still operates the service subject to public service obligation (market 1) and now also on market 2, without any constraint. Service in market 3 is supplied by a new firm $E$, which compete in prices with the incumbent.

The game takes place as follows: in the first stage, the incumbent learns the level of demand in market 2, i.e. he observes $\theta$ and chooses the
suitable level of subsidy offered by the regulator. The regulator publicly announces the value of \( \theta \). In the second stage, firms compete over prices.

We start solving the game under complete information, so that both the Transportation Authority and the firms know the demand in all markets.

In the second stage, firms compete in prices. As the incumbent provides services 1 and 2, it maximizes the total utility of the two segments including the lump sum transfer \( T \) decided in the first period:

\[
U^I = U_1 + U_2 + T
\]  

(5.)

or \( U^I = (p_1 - c_1)q_1 + (p_2 - c_2)q_2 + T \)

The other firm \( E \) operates only in market 3, so that his utility function is:

\[
U^E = U_3
\]  

(6.)

or \( U^E = (p_3 - c_3)q_3 \).

The reaction functions are:

\[
\begin{align*}
(p_1 - c_1) \frac{\partial q_1}{\partial p_2} + (p_2 - c_2) \frac{\partial q_2}{\partial p_2} + q_2 &= 0 \\
(p_3 - c_3) \frac{\partial q_3}{\partial p_3} + q_3 &= 0
\end{align*}
\]  

(7.)

The reaction function of the multiproduct firm \( I \) is directly affected by the level of per unit subsidy \( (p_1 - c_1) \), while the effect on market 3 is indirect, as it happens through the price of market 2.

In the first stage, the regulator defines the level of transfer that ensures the incumbent has a non negative utility \( U^I \geq 0 \). Under complete information, the regulator chooses \( T \) such that \( U^I = 0 \):

**Proposition 2:**

If a multiproduct firm \( I \), subject to PSO and active in market 2, competes in prices with another firm \( E \) active in market 3, the equilibrium prices are such that:
\[
\frac{p_2 - c_2}{p_2} = \frac{1}{\varepsilon_2} + \frac{\varepsilon_{12}}{\varepsilon_2} \frac{(p_1 - c_1)q_1}{p_2 q_2}
\]

(8.)

\[
\frac{p_3 - c_3}{p_3} = \frac{1}{\varepsilon_3}
\]

(9.)

The transfers \( T \) are such that \( U^I = U_1 + U_2 + T = 0 \). \hspace{1cm} (10.)

Recall that there exists imperfect substitutability among services, therefore the elasticities in expressions (8.) and (9.) are affected by prices in each market. For example, condition (9.) refers to the elasticity of the residual demand for market 3.

As underlined for the multiproduct regulated firm, the price is lower than it would be without that constraint.

### 2.3. PSO fund and competition

One of the proposed solutions to the financing of PSO is the set up of a fund, in which all the operators of the industry address part of the profits they obtain on competitive markets. This mechanism exists for example for telecommunications and for the postal sector, but it has not been applied to the railways.

In the first stage, the regulator defines the share of profits \( k \) and the lump sum transfer \( T \) for the operator in charge of PSO.

In the next period, the prices for service 2 and 3 are determined by the competition between firms.

As usual, we solve the problem backward, starting from the competitive outcome in the second period.

The utility function of the incumbent is given not only by the profits of the markets in which he is active, but also from the value of the PSO fund:

\[
PSOFund = k(p_2 - c_2)q_2 + k(p_3 - c_3)q_3 + T
\]

(11.)

So, the incumbent maximizes:

\[
U^I = U_1 + (1 - k)U_2 + PSOFund
\]

(12.)
that simplifies to $U^I = (p_1 - c_1)q_1 + (p_2 - c_2)q_2 + k(p_3 - c_3)q_3 + T$.

The entrant operates only on market 3, so that he maximizes:

$$U^E = (1 - k)U_3$$

or $U^E = (1 - k)(p_3 - c_3)q_3$

Only the reaction function of the incumbent is directly affected by $k$ as he internalises the share of profit that the entrant grants to the fund, as you can see from expressions in (14):

$$\begin{align*}
(p_1 - c_1) \frac{\partial q_1}{\partial p_2} + (p_2 - c_2) \frac{\partial q_2}{\partial p_2} + q_2 + k(p_3 - c_3) \frac{\partial q_3}{\partial p_2} &= 0 \\
(p_3 - c_3) \frac{\partial q_3}{\partial p_3} + q_3 &= 0
\end{align*}$$

In the first stage, the regulator has to define the optimal share of profits and the transfer $T$, maximizing the total welfare:

$$S(q_1) - p_1q_1 + S(q_2) - p_2q_2 + S(q_3) - p_3q_3 + U^I + U^E - (1 + \lambda)T$$

subject to the participation constraint of the incumbent: $U^I \geq 0$.

Rewriting the objective function and substituting for $T$, we obtain:

$$S(q_1) + S(q_2) + S(q_3) - (1 + \lambda)(c_1q_1 + c_2q_2) - (1 + \lambda)k(c_3q_3 +$$

$$+ \lambda(p_1q_1 + p_2q_2) + kp_3q_3) - \lambda U^I$$

where $U^I = 0$, as the game is under complete information.

The optimal $k = k^*$ is obtained by the derivative of (15.) computed for the equilibrium outcome.

**Proposition 3:**

Consider the case of a PSO fund where firms pay a share $k$ of the profits they obtain from competitive markets. If a multiproduct firm I, subject to PSO and active in market 2, competes in prices with firm E active in market 3, the equilibrium prices are such that:

$$\frac{p_2 - c_2}{p_2} = \frac{1}{\epsilon_2} + \frac{\epsilon_{1,2}}{\epsilon_2} (p_1 - c_1)q_1 + \frac{\epsilon_{3,2}}{\epsilon_3\epsilon_2} k^* p_3q_3$$

$$\frac{p_2 - c_2}{p_2} = \frac{1}{\epsilon_2} + \frac{\epsilon_{1,2}}{\epsilon_2} (p_1 - c_1)q_1 + \frac{\epsilon_{3,2}}{\epsilon_3\epsilon_2} k^* p_3q_3$$
\[
\frac{p_3 - c_3}{p_3} = \frac{1}{\varepsilon_3}
\]  
(17.)

\(k^*\) is the optimal share of profits determined maximizing the welfare function and is defined by the following condition:

\[
(1 + \lambda)(p_1 - c_1) \frac{\partial q_1}{\partial k} + (1 + \lambda)(p_2 - c_2) \frac{\partial q_2}{\partial k} + \\ + (1 + \lambda k)(p_3 - c_3) \frac{\partial q_3}{\partial k} + \lambda(p_3 - c_3)q_3 + \lambda k q_3 \frac{\partial p_3}{\partial k} + \lambda q_2 \frac{\partial p_2}{\partial k} = 0
\]  
(18.)

The transfers \(T\) are such that

\[
U^I \equiv U_1 + (1 - k)U_2 + PSOFund = 0
\]  
(19.)

The value \(k^*\) that solves the previous first order condition in (18.) is the percentage of the profit that each operator has to pay into the PSO fund. We consider only the cases in which demand and cost functions satisfy \(k^* \in (0,1)\), otherwise the use of PSO fund would not be feasible for the regulator.

In fact, if \(k^* = 0\) then there is no need for PSO fund and if \(k^* = 1\), it would not be socially feasible, as it would appear as an expropriation of all profits\(^{21}\). Therefore, we restrict our attention to the case in which \(k^* \in (0,1)\).

It is interesting to notice that even if a profit tax is usually considered not distortionary, this is not the case here. In fact, for the incumbent the contribution to the fund is an internal transfer between two sectors of his activity that leaves his utility unaffected. But then he also gains from the share of profit of the entrant. Finally, as there exists a link of substitutability between services, also the choice of the entrant is affected by \(k\) through the incumbent price.

\(\)  

\(^{21}\) With unconstrained optimization, if \(k^* < 0\), then the regulator is subsidizing the competitive markets, but this case is ruled out by law and if \(k^* > 1\), then the entrant’s utility would be lower than zero, so that it can’t be a solution.
3. Comparison of equilibrium outcomes under complete information

In this section we compare the different frameworks presented, using the demand functions in (a) to (c).

In order to perform the analysis, we compute the equilibrium outcomes according to some parameters chosen in such a way that it is possible to obtain positive prices and quantities in all market structures and that the characteristics of the market calls for the need of transfers\(^{22}\).

For example, referring to the intercept \(a_i\) we choose market 1 under PSO to be the biggest one, market 2 and 3 are respectively one half and one third of PSO market.

Moreover, we consider differences in the marginal costs to operate the services and different values of unit subsidy \((p_1 - c_1)\).

As for prices, it is possible to obtain a hierarchy of the frameworks (see for example Figure 13).

\[\text{Figure 13: prices for different values of the parameter } \theta\]

\(^{22}\) Numerical values of some of the simulations performed are in the Appendix A.2.2.
If the sector is entirely regulated, so that the Transportation Authority decides the prices for market 2 and 3, together with fixing the prices for PSOs, then those prices are the lowest.

From the equilibrium conditions we understood that the constraint on market 1 leads to a distortion of the prices in the other markets, but the downward distortion with a multiproduct regulated firm is larger than in the frameworks with competition, so that prices are lower.

On the other hand, when firms contribute to the PSO fund, they choose higher prices. The value of the participation share positively affects the reaction function of the incumbent directly so that he chooses higher prices, then in the absence of the fund and as services are substitutes, then also the price of market 3 increases.

Consider also that the regulator has to pay for transfer to the firm providing service under PSOs. When the PSO fund is implemented, the transfers required are smaller than in the other two frameworks (Figure 14 and Figure 15).

![Transfers](image)

**Figure 14:** Transfers for different values of $\theta$: $p_1 = 0.30c_1$
The comparison between the transfers in the multiproduct regulated firm and the competing multiproduct firm does not lead to a definite result as it depends on the share of cost covered with the price.

Define \( x = \frac{p_1}{c_1} \), \( x \in (0,1) \) the share of cost covered with the price, then there exists \( \tilde{x} \), such that for \( x > \tilde{x} \), the transfers under multiproduct regulation are lower, while if \( x < \tilde{x} \), the opposite is true.

A low \( x \) corresponds to a low price on PSO market, that is, it corresponds to a high per unit subsidy \( (p_1 - c_1) \).

The higher is this subsidy, the lower is the distortion induced on the other markets in both frameworks at glance. But the increase in prices on market 2 and 3 leads to higher quantity \( q_1 \) in market under PSO, so that the need for transfer increases. The point is that the for \( x < \tilde{x} \), \( q_1 \) increase is larger with a multiproduct regulated monopolist than when the incumbent has to compete with another firm, so that the transfers needed are larger in the first case than in the latter one.

We computed the total welfare and it appears that it is higher when the PSO is financed using the fund. Moreover, also the introduction of

**Figure 15:** Transfers for different values of \( \theta \): \( p_1 = 0.35c_1 \)
competition enhances welfare compared to the regulated framework (Figure 16).

![Figure 16: Total welfare for different values of $\theta$.](image)

Recall that we are considering the total welfare: even if the prices are lower if the services are provided by a multiproduct regulated monopolist, the welfare in the other two frameworks is higher because by construction of the model, the regulator cannot set the entrant’s utility at zero as it happens if there is a single firm. Moreover, there is less need for transfers and this implies a saving as they are costly.

To sum up, denote A the framework with multiproduct regulated monopolist, B the scenario with multiproduct firm and a competitor, C the case for PSO fund.

**Result 1:**

*Given $p_1$, the price for the service subject to public service obligation, then:*

(i) $p_{iA} < p_{iB} < p_{iC}$, with $i = 2, 3$

(ii) $T_C < T_A$ and $T_C < T_B$

(iii) $T_A < T_B$ iff $x > \bar{x}$
In the next section we consider the market structures illustrated above in case of asymmetric information over demand.

4. Market structures under asymmetric information

In the previous section we studied the effect on welfare of three possible kind of organization for a sector in which one of the firm has to fulfil public service obligations.

In what follows, we consider the same scenarios when the incumbent firm is the only one knowing the demand in market 2. In particular, there is asymmetric information over the parameter $\theta$. The regulator knows that $\theta$ is distributed according to the density function $f(\theta)$ in $[\underline{\theta}, \bar{\theta}]$, with cumulative distribution $F(\theta)$.

The inability to observe or compute the true value of $q_2$ affects both the choice of the regulator (prices, transfers, share of profits for the PSO fund) but also the choice of the entrant $E$ as he cannot observe $\theta$ too and relies on the regulator announcement in both competitive frameworks.

4.1. Multiproduct regulated monopolist

Under asymmetric information, neither the Transportation Authority nor the entrant know the level of the demand in market 2. Therefore she has to design a regulatory mechanism that induces the firm providing the service to reveal the level of demand.

The incumbent observes $\theta$ and chooses among the menu of contracts composed of prices and transfers.

The regulator maximizes the expected profit

$$\int_{\underline{\theta}}^{\bar{\theta}} \sum_{i=1}^{3} [S(q_i) - (1 + \lambda)c_i q_i + \lambda q_i p_i - \lambda U_i] f(\theta) d(\theta)$$

subject to the incentive compatibility constraint and the participation constraint.
The first constraint grants that the firm tells the truth as the utility it obtains is larger then the utility obtained reporting a different $\theta$:

$$U = (p_2 - c_2) \frac{\partial q_2}{\partial \theta}$$  \hspace{1cm} (21.)

The participation constraint ensures that also when demand in low, the firm is willing to participate to the market:

$$U(\theta) = 0$$ \hspace{1cm} (22.)

Therefore, the regulator maximizes:

$$\tilde{\theta} \int \sum_{i=1}^{3} \left[ S(q_i) - (1 + \lambda) c_i q_i + \lambda q_i p_i \right] f(\theta) - \lambda (1 - F(\theta)) (p_2 - c_2) \frac{\partial q_2}{\partial \theta} \right] d(\theta)$$

The solution for $p_2$ and $p_3$ are as those determined under complete information but also accounting for a downward information distortion. Only if the demand is at its maximum $\tilde{\theta}$, then there is no distortion.

**Proposition 4:**

Consider the case of asymmetric information over $\theta$.

If all services are provided by a multiproduct regulated monopolist, when there exists a PSO on market 1, the prices in market 2 and 3 are downward distorted relative to those under complete information (3.) and (4.).

The distortion $\Lambda_i$ induced by asymmetric information in market $i$ is:

$$\Lambda_i \equiv (-1)^i \frac{1}{p_i} \frac{\partial q_j}{\partial p_3} \frac{\lambda}{(1 + \lambda)} \frac{1 - F(\theta)}{f(\theta)} \frac{1}{\frac{\partial q_2}{\partial \theta}} \frac{\partial q_2}{\partial \theta} < 0$$  \hspace{1cm} (23.)

The incumbent’s rent is

$$\int_{\theta}^{\tilde{\theta}} \left[ (p_2 - c_2) \frac{\partial q_2}{\partial \theta} \right] d\theta$$ \hspace{1cm} (24.)
4.2. Multiproduct firm and single product competitor

Under asymmetric information, neither the regulator, nor the competitor know the value of the parameter $\theta$ that characterizes market 2.

Recall that in this framework, the regulator does not have a mechanism at hand that leads to truthful revelation of $\theta$.

Therefore, in the first stage, the Transportation Authority asks the incumbent for the level of demand in market 2 to define the lump sum transfer $T$ and announces $\theta$ to the entrant, who will decide his price according to it. We denote this value as $\hat{\theta}$.

In the second stage firms compete in prices: the entrant reacts according to his best response on the basis of the $\hat{\theta}$ announced by the regulator, while the incumbent also accounts for the true value of demand if it is different from the $\hat{\theta}$ reported to the Authority.

Define $q_2^{\hat{\theta}}$, $p_2$ the quantity and price in market 2 at $\hat{\theta}$.

The competitor’s choice is:

\[ p_3 = p_3(\hat{p}_2(\hat{\theta})) \] \hspace{1cm} (25.)

while the incumbent’s reaction function is:

\[ p_2 = p_2(\theta, p_3(\hat{\theta})) \] \hspace{1cm} (26.)

And the equilibrium outcome is given by the intersection of these reaction functions.

In the first stage, the incumbent reports to the regulator $\hat{\theta}$, such that his second period utility is maximized:

**Proposition 5:**

*If the incumbent is the only one knowing the value of $\theta$ and he competes with an entrant, then he asks for a level of transfers $T(\hat{\theta})$ that maximizes his utility knowing that $p_3 = p_3(\hat{p}_2(\hat{\theta}))$. Define $\bar{\theta} = \arg \max U^I$, then $\hat{\theta}$ is:*
Transfers are such that $U^I(\hat{\theta}) = 0$ (28.)

### 4.3. PSO fund

The last scheme we want to examine is the set up of the PSO fund. Recall that the timing of the game is as follows: in the first stage the incumbent chooses among the menu of contracts made up of the share of contribution to the PSO fund and the transfers. The regulator publicly announces the value of $\theta$, so that in the second period firms compete in prices according to that announcement and regulation is enforced.

Therefore, the competitor computes his best response function on the basis of $\hat{\theta}$ announced by the regulator.

As usual we solve the game backward.

Define $\hat{q}_2, \hat{p}_2$ the quantity and price in market 2 given the value of demand reported $\hat{\theta}$.

The competitor’s best response is:

$$p_3 = p_3(\hat{p}_2(\hat{\theta}))$$ (29.)

while the incumbent reacts according to $\hat{\theta}$ and $\theta$:

$$p_2 = p_2(\theta, p_3(\hat{\theta}))$$ (30.)

And the equilibrium outcome is given by the intersection of these reaction functions.

In the first period\(^{23}\), the regulator proposes a menu of contract to the incumbent $I$, designed to maximize the total expected welfare (28.) with respect to $k$ and $T$:

---

\(^{23}\) To safe notation, we do not denote the equilibrium outcome in a particular manner, but all the variables are those of second period competition.
The maximization is subject to the incentive constraint that ensures truthtelling, $U^I(\theta) \geq U^I(\hat{\theta})$:

$$U^I = (p_1 - c_1) \frac{\partial q_1}{\partial \theta} + (p_2 - c_2) \frac{\partial q_2}{\partial \theta} + k(p_3 - c_3) \frac{\partial q_3}{\partial \theta}$$

Using our demand specification, we can check that the incentive constraint is positive.

Moreover, the regulator also has to ensure that the participation constraint is satisfied also for the lowest value of demand: $U(\theta) = 0$

The regulator maximizes:

$$\int_{\theta} \left[ S(q_1) + S(q_2) + S(q_3) - (1 + \lambda)(c_1 q_1 + c_2 q_2) - (1 + \lambda k) c_3 q_3 + \lambda(p_1 q_1 + p_2 q_2) + \lambda k p_3 q_3 - \lambda U^I \right] d\theta$$

Considering the second period outcome, the share $k$ that satisfies the first order condition is downward distorted with respect to that defined under complete information:

**Proposition 6:**

*If the incumbent is the only one knowing the value of $\theta$ and the PSO fund is implemented,*

(i.) the optimal share of profit to be addressed to the PSO fund is downward distorted w.r.t. the complete information case and the distortion is

$$- \lambda \frac{(1 - F(\theta)) \frac{\partial U^I}{\partial k}}{f(\theta)}$$

(ii.) the equilibrium prices are also lower than under complete information

(iii.) the incumbent’s rent is

$$\int_{\theta} \left[ (p_1 - c_1) \frac{\partial q_1}{\partial \theta} + (p_2 - c_2) \frac{\partial q_2}{\partial \theta} + k(p_3 - c_3) \frac{\partial q_3}{\partial \theta} \right] d\theta$$
5. Comparison of equilibrium outcomes under asymmetric information

In order to compare the equilibrium outcomes under asymmetric information we compute prices, transfers and total welfare in the different market structures. The demand functions are (a) to (c).

As for the complete information case, we choose the parameters in order to have feasible outcomes for prices and quantities. Moreover, the characteristic of the markets are such that there is the need for transfers to cover market 1 deficit (the intercept for market 2 and 3 are approximately one forth on the PSO market). The parameter $\theta$ is observed only by the incumbent and the regulator only knows that is randomly drawn from a uniform distribution in the interval $[1,2]$.

The prices chosen by the regulator, if the monopolist is entirely regulated are still the lowest then the prices of the other two frameworks.

The implementation of the PSO fund increases the prices in the competitive markets, so that under this market structure consumers pay more (Figure 17)

![Prices in Market 2](image)

**Figure 17:** Prices for different values of $\theta$

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24 Numerical values of some of the simulations performed are in the Appendix A.2.2.
Differently from what obtained under complete information, there is no longer an issue about which market structure requires the lower level of transfers (see Figure 18). We saw that depending on the level of unit subsidy to the PSO, it could happen that the transfers for a multiproduct regulated monopolist could be lower or higher than those required in case of competition. With asymmetric information, the transfers are always lower under monopoly and this is due to the absence of an incentive mechanism that induces thoroughful revelation of $\theta$. Moreover, PSO fund keep on being the structure with the lowest level of transfers.

![Figure 18: Transfers for different values of $\theta$](image)

As for total welfare (Figure 19), it appears that the implementation of the PSO fund grants higher welfare than the other frameworks.
Figure 19: **Total welfare for different values of \( \theta \)**

To sum up the results we obtained, denote A the multiproduct regulated monopoly, B the scenario with multiproduct firm and a competitor, C the case for PSO fund.

**Result 2:**

*Given \( p_1 \), the price for the service subject to public service obligation, then:*

(i) \( p_{iA} < p_{iB} < p_{iC} \), with \( i = 2,3 \)

(ii) \( T_C < T_A < T_B \)

(iv) \( W_A < W_B < W_C \)

**6. The share of participation to PSO fund**

In this paper, the PSO fund mechanism requires both firms contributing with the same share of profit and in particular, we considered \( k \) as the optimal share from the social point of view.

But in the telecommunications and postal sector, where the PSO fund is already active, the share of contribution is decided according to a measure of the market share of each firm.

Therefore, it is interesting to understand how prices are affected by the rule chosen to determine the percentage of profit to pay.
Moreover, it is also important at which stage \( k \) is determined. If \( k \) is known only after competition has taken place, it is the case that firms internalize the effect of prices on \( k \) when they compete.

We briefly consider this issue focusing on the effect of \( k \) on market 2 and 3, as competition takes place in these markets.

The demand function of each service is linear and the services are imperfect substitutes. As for the cost function, there are no fixed costs and the marginal cost is constant.

The share \( k_i \) in market \( i \) can be defined before or after the competition. We refer to the first case as ante competition and to the second as post competition.

Moreover, \( k_i \) is computed either on the basis of revenues or of quantities. We first consider it as the ratio between the quantity of the firm we are considering (firm \( i \)) and the total quantity of the competitive segment and then as the ratio between revenues.

When \( k \) is defined ex ante, the timing of the game is the following: in the first stage, the regulator announces \( k \) and a lump sum transfer \( T \) for the provider of PSOs; in the second stage firms compete in prices.

On the other hand, if \( k \) is defined ex post, then in the first stage, the regulator announces the rule to define \( k \) and the transfer \( T \). Firms compete in prices. In the second stage the regulator asks for contribution into PSO fund, according to the rule for \( k \).

Define the utility function of the firms as:

\[
U^I = U_2 + k_3 U_3 + T \\
U^E = (1 - k_3)U_3
\]

in which \( U_i \) is the utility of the firm in market \( i \).

Therefore, the reaction functions of the firms are such that for the incumbent

\[
\frac{\partial U_2}{\partial p_2} + k_3 \frac{\partial U_3}{\partial p_2} + U_3 \frac{\partial k_3}{\partial p_2} = 0 \tag{36.}
\]

and for the competitor
If \( k \) is the ratio between quantities, then \( k_3 \) is decreasing in \( p_3 \) and increasing in \( p_2 \).

On the other hand, if \( k \) is computed according to the revenues, then \( k_3 \) is still increasing in \( p_2 \), but it is not possible to define the sign of the effect of \( p_3 \) on \( k_3 \).

Therefore, comparing the equilibrium prices when \( k \) is decided before or after competition gives a definite answer only if the share of participation to the PSO fund is computed on the basis of quantities.

**Result 1:** If the share of participation to the PSO fund is defined as the ratio between quantities after competition has taken place, then firms compete less aggressively then when \( k \) is defined before competition.

If \( k \) is decided according to revenues, then prices could be either higher or lower.

This result highlights that both the timing and the rule to determine the share of participation to the PSO fund has to be carefully considered, because as explained above deciding it before or after competition obviously affects the equilibrium outcome. More important, if the rule is the ratio between quantities, then the effect is clear: firms choose higher prices to detriment of consumers. But if \( k \) is computed on the basis of revenues, then the effect is not clear and depends on demand and cost functions.

7. Concluding remarks and possible extensions

In this paper we focus on the issue of how to organize the railway industry in order to satisfy the reform requirements and keep on providing services belonging to PSO.

We compared three market structures: a multiproduct regulated monopolist, the introduction of competition without any regulation except PSO and finally the set up of a PSO fund.
In order to have an idea about the effect of these structures on the total welfare we computed equilibrium outcomes according to different values of the parameters.

It appears that even if the multiproduct regulated monopolist leads to lower prices, the total welfare is higher when a PSO fund is implemented. This result is related to the lower level of transfers required to cover the PSO deficit if all the providers contribute to the fund.

This result holds also under asymmetric information over demand in one of the market. The symmetry of information affects the comparison between the multiproduct regulated monopolist framework and the case of introduction of competition. In fact, while under complete information it is not clear which framework asks for lower transfers, under asymmetric information, if the regulator has only to pay for the PSO deficit, then the transfers required by the incumbent will be higher. This result is due to the fact that the regulator does not have any incentive device to induce truthful revelation of the demand.

An interesting extension is related to the budget constraint of the regulator. Instead of taking care of the entire utility of the incumbent, the regulator could also decide to grant only the transfers that cover PSO. There is an ongoing discussion about this point as according to some opinion it should be the case that the regulator only control that PSOs are fulfilled and that the PSO provider is not at disadvantage relative to the other provider. Obviously, other sustain the opposite, that is, the regulator should look at the total profit of the firm.
Conclusions

The reform of the railway industry calls for more specific analysis of the sector and of the effect of liberalization in this industry.

As highlighted at the beginning, even if a large number of studies propose qualitative analysis of the phenomenon, only a few adopt an empirical or theoretical approach.

The objective of this thesis was to give some insights about two special characteristics of railways, the problem of rolling stock and the existence of public service obligations.

The efforts of the European Union to promote competition do not fit with the evidence that in most of the countries the service is still provided only by the historical incumbent.

We identify in the difficulty to obtain rolling stock one of the possible explanation. The solutions adopted across countries vary from ROSCOs to public owned rolling stock rent to the provider.

The choice between private or public ownership of the rolling stock leads to different level of investment that have to be evaluated in light of some market conditions such as the cost of public funds and the effect of the investment on costs and rolling stocks. If the public authority does not have all the information about the costs of the firm, than the question of which framework grants higher investment does not have a clear cut answer.

We specified the model to study the probability of the incumbent to be selected and we found that the positive effect of the first period investment increases the probability for the incumbent to keep on being the provider of the service. This finding is in line with what we observe in many countries, including Italy: the company operating the line is very often the same that will be active for the next period contract.

Another interesting issue related to the opening of the railway market is how to manage public service obligation. The introduction of competition raises questions about allocation and funding of these obligations.

We took a different approach from the literature of universal service obligation, modelling the constraint imposed on a single firm as an exogenous given price, below marginal cost. Moreover, the link
between markets depends on the imperfect substitutability among services.

Comparing different market structures it appears that if the aim is to have public service obligations fulfilled by a firm operating also in at least another market, then the set up of a PSO fund grants gains in welfare and this is true both under complete and asymmetric information over demand.

We think that these issues worth more analysis especially in light of the ongoing changes in the sector and of the discretion left to countries that calls for deep studies about which solution fits best in which context.
Appendix

A.1. Appendix to Chapter 2

In this appendix we report some of the simulations we run to are the threshold defined by the regulator to choose the second period provider.

The model is specified as follows: we assume that the cost parameters of the firms are uniformly distributed over the interval \([1,2]\).

Moreover, the disutility of the investment is \(d(a_{t,j}^i) = (a_{t,j}^i)^2\) and the benefit on the rolling stock is \(b(a_{t,j}^i) = \frac{1}{5} a_{t,j}^i\).

As the ownership is public, the local authority pays a reward \(x(a_{t,PUB}^i) = \theta b(a_{t,PUB}^i)\) and gains \(u(a_{2,PUB}^i) = \omega b(a_{2,PUB}^i)\), where \(\theta, \omega \in [0,1]\).

For each simulation the table reports:

- value of \(\beta I\)
- the parameter whose effect we are observing
- the threshold under asymmetric information
- the difference between the threshold under asymmetric and under complete information

\[ \Delta_j = \beta_j^* - \beta_j^{**} \text{ (with } j = \text{ PR, PUB)} \]

- the first period investment \(a_{1,j}\)

- the second period investment \(a_{2,j}^I\) if the incumbent is active

- the second period investment \(a_{2,j}^C\) if a competitor matching the threshold is selected

- the probability for the incumbent to be selected as second period provider
a. Private Ownership

- Residual value

\[ \delta = 0.5; \quad \lambda = 0.1; \quad k = 0.1; \quad g = 0.1; \quad h = 0.1. \]

<table>
<thead>
<tr>
<th>( \beta^I )</th>
<th>R</th>
<th>Threshold asymmetric info</th>
<th>Delta threshold</th>
<th>( a_{1,PR} )</th>
<th>( a_{2,PR} )</th>
<th>( a_{2,PR}^C )</th>
<th>Probability of I to be selected</th>
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Cost of public funds

\[ R = 0.2; \quad \delta = 0.7; \quad k = 0.1; \quad g = 0.1; \quad h = 0.1. \]

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<th>( a_{2,PR} )</th>
<th>( a_{PR} )</th>
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<td>$a_{2,PR}^C$</td>
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</tr>
<tr>
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<td>-----------</td>
<td>---------------------------</td>
<td>-----------------</td>
<td>---------</td>
<td>-------------</td>
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</tr>
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</table>

- **Discount factor**

$R = 0.2; \quad \lambda = 0.4; \quad k = 0.1; \quad g = 0.1; \quad h = 0.1.$

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<th>Threshold asymmetric info</th>
<th>Delta threshold</th>
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<th>$a_{2,PR}^C$</th>
<th>Probability of I to be selected</th>
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<td>0.561</td>
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<td>0.536</td>
<td>0.522</td>
<td>72.69%</td>
</tr>
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### Threshold asymmetric info

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<th>Probability of $I$ to be selected</th>
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<td>0.320</td>
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<td>0.378</td>
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<tr>
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</table>

- **Transferable part of the investment**

$\delta = 0.5; \quad \lambda = 0.2; \quad k = 0.1; \quad R = 0.2; \quad h = 0.05.$

<table>
<thead>
<tr>
<th>$\beta^l$</th>
<th>$g$</th>
<th>Threshold asymmetric info</th>
<th>Delta threshold</th>
<th>$a_{1,PR}$</th>
<th>$a_{2,PR}^I$</th>
<th>$a_{2,PR}^C$</th>
<th>Probability of $I$ to be selected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>0.2</td>
<td>1.046</td>
<td>0.033</td>
<td>0.586</td>
<td>0.578</td>
<td>0.592</td>
<td>95.44%</td>
</tr>
<tr>
<td>1.4</td>
<td>0.2</td>
<td>1.145</td>
<td>0.030</td>
<td>0.565</td>
<td>0.570</td>
<td>0.576</td>
<td>85.45%</td>
</tr>
<tr>
<td>1.5</td>
<td>0.2</td>
<td>1.246</td>
<td>0.027</td>
<td>0.545</td>
<td>0.563</td>
<td>0.559</td>
<td>75.44%</td>
</tr>
<tr>
<td>1.6</td>
<td>0.2</td>
<td>1.346</td>
<td>0.025</td>
<td>0.525</td>
<td>0.555</td>
<td>0.542</td>
<td>65.41%</td>
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119
<table>
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<th>$\beta^I$</th>
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<th>Threshold asymmetric info</th>
<th>Delta threshold</th>
<th>$a_{1,PR}^I$</th>
<th>$a_{2,PR}^I$</th>
<th>$a_{2,PR}^C$</th>
<th>Probability of I to be selected</th>
</tr>
</thead>
<tbody>
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<td>1.446</td>
<td>0.022</td>
<td>0.504</td>
<td>0.548</td>
<td>0.526</td>
<td>55.36%</td>
</tr>
<tr>
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<td>0.020</td>
<td>0.484</td>
<td>0.540</td>
<td>0.509</td>
<td>45.29%</td>
</tr>
<tr>
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<td>1.648</td>
<td>0.017</td>
<td>0.463</td>
<td>0.533</td>
<td>0.492</td>
<td>35.20%</td>
</tr>
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<td>0.525</td>
<td>0.475</td>
<td>25.09%</td>
</tr>
<tr>
<td>1.3</td>
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<td>0.578</td>
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<td>0.570</td>
<td>0.580</td>
<td>87.79%</td>
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<tr>
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<td>0.028</td>
<td>0.554</td>
<td>0.563</td>
<td>0.563</td>
<td>77.76%</td>
</tr>
<tr>
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<td>0.024</td>
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<td>0.555</td>
<td>0.546</td>
<td>67.71%</td>
</tr>
<tr>
<td>1.7</td>
<td>0.4</td>
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<td>0.021</td>
<td>0.511</td>
<td>0.548</td>
<td>0.529</td>
<td>57.64%</td>
</tr>
<tr>
<td>1.8</td>
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<td>1.524</td>
<td>0.017</td>
<td>0.489</td>
<td>0.540</td>
<td>0.513</td>
<td>47.55%</td>
</tr>
<tr>
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<td>1.626</td>
<td>0.014</td>
<td>0.468</td>
<td>0.533</td>
<td>0.496</td>
<td>37.44%</td>
</tr>
<tr>
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<td>0.4</td>
<td>1.727</td>
<td>0.012</td>
<td>0.446</td>
<td>0.525</td>
<td>0.479</td>
<td>27.31%</td>
</tr>
<tr>
<td>1.4</td>
<td>0.6</td>
<td>1.098</td>
<td>0.033</td>
<td>0.585</td>
<td>0.570</td>
<td>0.584</td>
<td>90.20%</td>
</tr>
<tr>
<td>1.5</td>
<td>0.6</td>
<td>1.199</td>
<td>0.028</td>
<td>0.563</td>
<td>0.563</td>
<td>0.567</td>
<td>80.15%</td>
</tr>
<tr>
<td>1.6</td>
<td>0.6</td>
<td>1.299</td>
<td>0.024</td>
<td>0.540</td>
<td>0.555</td>
<td>0.550</td>
<td>70.07%</td>
</tr>
<tr>
<td>1.7</td>
<td>0.6</td>
<td>1.400</td>
<td>0.019</td>
<td>0.518</td>
<td>0.548</td>
<td>0.533</td>
<td>59.98%</td>
</tr>
<tr>
<td>1.8</td>
<td>0.6</td>
<td>1.501</td>
<td>0.015</td>
<td>0.495</td>
<td>0.540</td>
<td>0.516</td>
<td>49.86%</td>
</tr>
<tr>
<td>1.9</td>
<td>0.6</td>
<td>1.603</td>
<td>0.011</td>
<td>0.473</td>
<td>0.533</td>
<td>0.500</td>
<td>39.73%</td>
</tr>
<tr>
<td>2</td>
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<td>1.704</td>
<td>0.008</td>
<td>0.450</td>
<td>0.525</td>
<td>0.483</td>
<td>29.57%</td>
</tr>
</tbody>
</table>
- Non transferable part of the investment

\[ \delta = 0.5; \quad \lambda = 0.2; \quad k = 0.1; \quad R = 0.2; \quad g = 0.05. \]

<table>
<thead>
<tr>
<th>( \beta^I )</th>
<th>( h )</th>
<th>Threshold asymmetric info</th>
<th>Delta threshold</th>
<th>( a_{1,PR} )</th>
<th>( a_{2,PR} )</th>
<th>( a_{2,PR}^C )</th>
<th>Probability of I to be selected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>0.2</td>
<td>1.046</td>
<td>0.033</td>
<td>0.586</td>
<td>0.578</td>
<td>0.592</td>
<td>95.44%</td>
</tr>
<tr>
<td>1.4</td>
<td>0.2</td>
<td>1.145</td>
<td>0.030</td>
<td>0.565</td>
<td>0.570</td>
<td>0.576</td>
<td>85.45%</td>
</tr>
<tr>
<td>1.5</td>
<td>0.2</td>
<td>1.246</td>
<td>0.027</td>
<td>0.545</td>
<td>0.563</td>
<td>0.559</td>
<td>75.44%</td>
</tr>
<tr>
<td>1.6</td>
<td>0.2</td>
<td>1.346</td>
<td>0.025</td>
<td>0.525</td>
<td>0.555</td>
<td>0.542</td>
<td>65.41%</td>
</tr>
<tr>
<td>1.7</td>
<td>0.2</td>
<td>1.446</td>
<td>0.022</td>
<td>0.504</td>
<td>0.548</td>
<td>0.526</td>
<td>55.36%</td>
</tr>
<tr>
<td>1.8</td>
<td>0.2</td>
<td>1.547</td>
<td>0.020</td>
<td>0.484</td>
<td>0.540</td>
<td>0.509</td>
<td>45.29%</td>
</tr>
<tr>
<td>1.9</td>
<td>0.2</td>
<td>1.648</td>
<td>0.017</td>
<td>0.463</td>
<td>0.533</td>
<td>0.492</td>
<td>35.20%</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>1.749</td>
<td>0.016</td>
<td>0.443</td>
<td>0.525</td>
<td>0.475</td>
<td>25.09%</td>
</tr>
<tr>
<td>1.3</td>
<td>0.4</td>
<td>1.022</td>
<td>0.035</td>
<td>0.596</td>
<td>0.578</td>
<td>0.596</td>
<td>97.80%</td>
</tr>
<tr>
<td>1.4</td>
<td>0.4</td>
<td>1.122</td>
<td>0.031</td>
<td>0.575</td>
<td>0.570</td>
<td>0.580</td>
<td>87.79%</td>
</tr>
<tr>
<td>1.5</td>
<td>0.4</td>
<td>1.222</td>
<td>0.028</td>
<td>0.554</td>
<td>0.563</td>
<td>0.563</td>
<td>77.76%</td>
</tr>
<tr>
<td>1.6</td>
<td>0.4</td>
<td>1.323</td>
<td>0.024</td>
<td>0.532</td>
<td>0.555</td>
<td>0.546</td>
<td>67.71%</td>
</tr>
<tr>
<td>1.7</td>
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<td>1.424</td>
<td>0.021</td>
<td>0.511</td>
<td>0.548</td>
<td>0.529</td>
<td>57.64%</td>
</tr>
<tr>
<td>1.8</td>
<td>0.4</td>
<td>1.524</td>
<td>0.017</td>
<td>0.489</td>
<td>0.540</td>
<td>0.513</td>
<td>47.55%</td>
</tr>
<tr>
<td>1.9</td>
<td>0.4</td>
<td>1.626</td>
<td>0.014</td>
<td>0.468</td>
<td>0.533</td>
<td>0.496</td>
<td>37.44%</td>
</tr>
<tr>
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<td>0.4</td>
<td>1.727</td>
<td>0.012</td>
<td>0.446</td>
<td>0.525</td>
<td>0.479</td>
<td>27.31%</td>
</tr>
<tr>
<td>1.4</td>
<td>0.6</td>
<td>1.098</td>
<td>0.033</td>
<td>0.585</td>
<td>0.570</td>
<td>0.584</td>
<td>90.20%</td>
</tr>
<tr>
<td>1.5</td>
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<td>0.028</td>
<td>0.563</td>
<td>0.563</td>
<td>0.567</td>
<td>80.15%</td>
</tr>
<tr>
<td>1.6</td>
<td>0.6</td>
<td>1.299</td>
<td>0.024</td>
<td>0.540</td>
<td>0.555</td>
<td>0.550</td>
<td>70.07%</td>
</tr>
<tr>
<td>1.7</td>
<td>0.6</td>
<td>1.400</td>
<td>0.019</td>
<td>0.518</td>
<td>0.548</td>
<td>0.533</td>
<td>59.98%</td>
</tr>
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</table>
### Table 1: Effect on second period cost

<table>
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<tr>
<th>$\beta^I$</th>
<th>$h$</th>
<th>Threshold asymmetric info</th>
<th>Delta threshold</th>
<th>$a_{1,PR}$</th>
<th>$a_{1,PR}^I$</th>
<th>$a_{2,PR}^C$</th>
<th>Probability of $I$ to be selected</th>
</tr>
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<tbody>
<tr>
<td>1.8</td>
<td>0.6</td>
<td>1.501</td>
<td>0.015</td>
<td>0.495</td>
<td>0.540</td>
<td>0.516</td>
<td>49.86%</td>
</tr>
<tr>
<td>1.9</td>
<td>0.6</td>
<td>1.603</td>
<td>0.011</td>
<td>0.473</td>
<td>0.533</td>
<td>0.500</td>
<td>39.73%</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>1.704</td>
<td>0.008</td>
<td>0.450</td>
<td>0.525</td>
<td>0.483</td>
<td>29.57%</td>
</tr>
</tbody>
</table>

- **Effect on second period cost**

  $\delta = 0.7; \quad \lambda = 0.2; \quad k = 0.1; \quad R = 0.2; \quad h = g = 0.1$

<table>
<thead>
<tr>
<th>$\beta^I$</th>
<th>$k$</th>
<th>Threshold asymmetric info</th>
<th>Delta threshold</th>
<th>$a_{1,PR}$</th>
<th>$a_{1,PR}^I$</th>
<th>$a_{2,PR}^C$</th>
<th>Probability of $I$ to be selected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>0.2</td>
<td>1.088</td>
<td>0.034</td>
<td>0.610</td>
<td>0.573</td>
<td>0.585</td>
<td>91.23%</td>
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<td>1.5</td>
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<td>1.189</td>
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<td>0.585</td>
<td>0.567</td>
<td>0.568</td>
<td>81.09%</td>
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<td>0.560</td>
<td>0.552</td>
<td>70.92%</td>
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<td>0.021</td>
<td>0.534</td>
<td>0.553</td>
<td>0.535</td>
<td>60.73%</td>
</tr>
<tr>
<td>1.8</td>
<td>0.2</td>
<td>1.495</td>
<td>0.017</td>
<td>0.509</td>
<td>0.547</td>
<td>0.518</td>
<td>50.51%</td>
</tr>
<tr>
<td>1.9</td>
<td>0.2</td>
<td>1.597</td>
<td>0.014</td>
<td>0.484</td>
<td>0.540</td>
<td>0.500</td>
<td>40.26%</td>
</tr>
<tr>
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<td>1.700</td>
<td>0.010</td>
<td>0.459</td>
<td>0.533</td>
<td>0.483</td>
<td>29.98%</td>
</tr>
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<td>0.592</td>
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</tr>
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<td>0.570</td>
<td>0.575</td>
<td>84.88%</td>
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<td>1.364</td>
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<td>0.565</td>
<td>0.560</td>
<td>0.539</td>
<td>63.56%</td>
</tr>
<tr>
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<td>1.472</td>
<td>0.005</td>
<td>0.531</td>
<td>0.555</td>
<td>0.521</td>
<td>52.83%</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
<td>1.579</td>
<td>-0.001</td>
<td>0.498</td>
<td>0.550</td>
<td>0.503</td>
<td>42.06%</td>
</tr>
<tr>
<td>$\beta^I$</td>
<td>k</td>
<td>Threshold asymmetric info</td>
<td>Delta threshold</td>
<td>$a_{1,PR}$</td>
<td>$a_{2,PR}$</td>
<td>$a_{2,PR}^C$</td>
<td>Probability of I to be selected</td>
</tr>
<tr>
<td>---------</td>
<td>----</td>
<td>---------------------------</td>
<td>-----------------</td>
<td>----------</td>
<td>----------</td>
<td>-------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>1.7</td>
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<td>0.586</td>
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</tr>
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<td>1.200</td>
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<td>0.646</td>
<td>0.573</td>
<td>0.567</td>
<td>79.97%</td>
</tr>
<tr>
<td>1.9</td>
<td>0.6</td>
<td>1.315</td>
<td>0.002</td>
<td>0.603</td>
<td>0.570</td>
<td>0.547</td>
<td>68.46%</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>1.431</td>
<td>-0.010</td>
<td>0.561</td>
<td>0.567</td>
<td>0.528</td>
<td>56.89%</td>
</tr>
</tbody>
</table>
b. Public Ownership

- Cost of public funds

\[ \delta = 0.7; \quad \theta = 0.5; \quad \omega = 0.3; \quad k = 0.1; \quad h = g = 0.1 \]

<table>
<thead>
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### Discount factor

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<th>( \lambda )</th>
<th>Threshold asymmetric info</th>
<th>Delta threshold</th>
<th>( a_{1,PUB} )</th>
<th>( a_{2,PUB} )</th>
<th>( a_{2,PUB}^C )</th>
<th>Probability of I to be selected</th>
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<td>$a_{2,PUB}$</td>
<td>$a_{C,PUB}$</td>
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- **Transferable part of the investment**
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<th>$\bar{g}$</th>
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<th>$a_{2,PUB}$^I</th>
<th>$a_{2,PUB}$^C</th>
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**Non transferable part of the investment**

\[ \lambda = 0.2; \quad \theta = 0.5; \quad \omega = 0.3; \quad k = 0.1; \quad g = 0.05 \]

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<th>( g )</th>
<th>Threshold asymmetric info</th>
<th>Delta threshold</th>
<th>( a_{1,PUB} )</th>
<th>( a_{2,PUB} )</th>
<th>( a_{2,C} )</th>
<th>Probability of I to be selected</th>
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<td>0.571</td>
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<td>0.554</td>
<td>67.29%</td>
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<tr>
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<td>0.537</td>
<td>57.23%</td>
</tr>
<tr>
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<tr>
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<td>0.601</td>
<td>0.608</td>
<td>99.62%</td>
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<tr>
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<td>0.593</td>
<td>0.591</td>
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<td>Threshold asymmetric info</td>
<td>Delta threshold</td>
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<td>$a_{2,PUB}^I$</td>
<td>$a_{2,PUB}^C$</td>
<td>Probability of I to be selected</td>
</tr>
<tr>
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<td>-----</td>
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<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------------------------</td>
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<td>0.595</td>
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<td>0.460</td>
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</table>
• **Reward**

\[ \lambda = 0.2; \quad \delta = 0.5; \quad \omega = 0.3; \quad k = 0.1; \quad h = g = 0.05 \]

<table>
<thead>
<tr>
<th>( \beta^I )</th>
<th>( \theta )</th>
<th>Threshold asymmetric info</th>
<th>Delta threshold</th>
<th>( a_{1.PUB} )</th>
<th>( a_{2.PUB}^I )</th>
<th>( a_{2.PUB}^C )</th>
<th>Probability of I to be selected</th>
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<td>1.043</td>
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<td>0.601</td>
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</tr>
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<td>1.143</td>
<td>0.006</td>
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<td>0.593</td>
<td>0.584</td>
<td>85.69%</td>
</tr>
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<td>0.2</td>
<td>1.243</td>
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<td>0.557</td>
<td>0.586</td>
<td>0.568</td>
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<tr>
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<td>0.578</td>
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<td>65.66%</td>
</tr>
<tr>
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<td>0.571</td>
<td>0.534</td>
<td>55.62%</td>
</tr>
<tr>
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<td>0.601</td>
<td>95.70%</td>
</tr>
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<td>0.576</td>
<td>0.593</td>
<td>0.585</td>
<td>85.71%</td>
</tr>
<tr>
<td>1.3</td>
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<td>0.586</td>
<td>0.568</td>
<td>75.70%</td>
</tr>
<tr>
<td>1.4</td>
<td>0.4</td>
<td>1.343</td>
<td>0.002</td>
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<td>0.578</td>
<td>0.551</td>
<td>65.67%</td>
</tr>
<tr>
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<td>0.518</td>
<td>0.571</td>
<td>0.534</td>
<td>55.62%</td>
</tr>
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<td>0.518</td>
<td>45.55%</td>
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<td>0.484</td>
<td>25.35%</td>
</tr>
<tr>
<td>1.9</td>
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<td>0.440</td>
<td>0.541</td>
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<td>15.23%</td>
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131
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<th>$\theta$</th>
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<th>Delta threshold</th>
<th>$a_{1,PUB}$</th>
<th>$a_{2,PUB}$</th>
<th>$a_{C,PUB}$</th>
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<td>75.73%</td>
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<td>0.571</td>
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<td>0.440</td>
<td>0.541</td>
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</table>

- Social utility

$\lambda = 0.2; \quad \delta = 0.5; \quad \theta = 0.5; \quad k = 0.1; \quad h = g = 0.05$

<table>
<thead>
<tr>
<th>$\beta^I$</th>
<th>$\omega$</th>
<th>Threshold asymmetric info</th>
<th>Delta threshold</th>
<th>$a_{1,PUB}$</th>
<th>$a_{2,PUB}$</th>
<th>$a_{C,PUB}$</th>
<th>Probability of I to be selected</th>
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<td>1.143</td>
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<td>0.576</td>
<td>0.585</td>
<td>0.576</td>
<td>85.73%</td>
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<td>75.71%</td>
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A.2. Appendix to Chapter 4

A.2.1. Market structures


c. Market structure A – Multiproduct monopoly

Complete information

The regulator maximizes the welfare function, with respect to prices of service 2 and 3, given \( p_1 - c_1 < 0 \)

\[
W_A = \sum_{i=1}^{3} [S(q_i) - p_i q_i] + U^I - (1 + \lambda) T
\]

Simplifies to:

\[
W_A = \sum_{i=1}^{3} [S(q_i) - (1 + \lambda) C_i(q_i) + \lambda p_i q_i] - \lambda U^I
\]

The first order conditions are:

\[
\begin{bmatrix}
\frac{\partial q_2}{\partial p_2} & \frac{\partial q_3}{\partial p_2} \\
\frac{\partial q_2}{\partial p_3} & \frac{\partial q_3}{\partial p_3}
\end{bmatrix}
\begin{bmatrix}
p_2 - c_2 \\
p_3 - c_3
\end{bmatrix}
= -\begin{bmatrix}
\frac{\partial q_1}{\partial p_2} (p_1 - c_1 ) + \frac{\lambda}{1 + \lambda} q_2 \\
\frac{\partial q_1}{\partial p_3} (p_1 - c_1 ) + \frac{\lambda}{1 + \lambda} q_3
\end{bmatrix}
\]

The result of the above system is

\[
\frac{p_i - c_i}{p_i} = \frac{1}{\epsilon_i} \frac{\epsilon_{j,i} q_j p_j}{1 + \lambda} \frac{\epsilon_j q_i p_i}{\epsilon_i} + \frac{\epsilon_{1,i} (p_1 - c_1) q_1}{\epsilon_i} \frac{1}{1 + \lambda} \frac{\epsilon_{j,i} q_j p_j}{\epsilon_i} + \frac{\epsilon_{1,i} \epsilon_{j,i}}{\epsilon_i \epsilon_j} \frac{1}{1 + \lambda} \frac{\epsilon_{j,i} q_j p_j}{\epsilon_i} + \frac{\epsilon_{1,i} \epsilon_{j,i}}{\epsilon_i \epsilon_j}
\]

Asymmetric information

Under asymmetric information the regulator maximizes the expected profit

\[
\bar{\theta} \sum_{i=1}^{3} [S(q_i) - (1 + \lambda) C_i(q_i) + \lambda q_i p_i - \lambda U_i] f(\theta) d(\theta)
\]

subject to the incentive compatibility constraint and the participation constraint.
The first constraint grants that the firm tells the truth as the utility it obtains is larger then the utility obtained reporting a different $\theta$:

$$\dot{U} = (p_2 - c_2) \frac{\partial q_2}{\partial \theta}$$

while the participation constraint ensures that also when demand in low, the firm is willing to participate to the market:

$$U(\theta) = 0$$

To solve this problem we use $p_2$ as control variable, $U$ as state variable and define the costate variable $\mu$. The Hamiltonian is:

$$H = W + \mu \dot{U}$$

The value of the costate variable is defined by $\dot{\mu} = -\frac{\partial H}{\partial U} = \lambda f(\theta)$ and the transversality condition at $\mu(\bar{\theta}) = 0$: $\mu = \lambda(F(\theta) - 1)$.

Therefore, the regulator maximizes:

$$\int_{\theta}^{\bar{\theta}} \left[ \sum_{i=1}^{3} [S(q_i) - (1 + \lambda)C_i(q_i) + \lambda q_i p_i]f(\theta) - \lambda(1 - F(\theta))(p_2 - c_2) \frac{\partial q_2}{\partial \theta} \right] d(\theta)$$

The solution for $p_2$ and $p_3$ are as those determined under complete information but also accounting for a downward information distortion. For market 2, the distortion is:

$$\frac{1}{p_2} \frac{\lambda}{(1 + \lambda) \left( \frac{\partial q_2}{\partial p_2} \frac{\partial q_3}{\partial p_3} - \frac{\partial q_3}{\partial p_2} \frac{\partial q_2}{\partial p_3} \right)} \frac{1 - F(\theta)}{f(\theta)} \frac{\partial q_2}{\partial \theta} \frac{\partial q_3}{\partial p_3} < 0$$

And for market 3, the distortion is:

$$-\frac{1}{p_3} \frac{\lambda}{(1 + \lambda) \left( \frac{\partial q_2}{\partial p_2} \frac{\partial q_3}{\partial p_3} - \frac{\partial q_3}{\partial p_2} \frac{\partial q_2}{\partial p_3} \right)} \frac{1 - F(\theta)}{f(\theta)} \frac{\partial q_2}{\partial \theta} \frac{\partial q_2}{\partial p_3} < 0$$

In both cases there is a downward distortion with respect to complete information. As $\frac{1 - F(\theta)}{f(\theta)}$ increases in $\theta$, at $\bar{\theta}$ there is no distortion.
Market structure B – Multiproduct firm + single product competitor for service 3

Complete information

The regulator sets $p_1 = c_1 + \delta$ and the price for services 2 and 3 are determined by the competition over prices between $I$ and $E$.

The incumbent maximizes its utility function:

$$U^I = (p_1 - c_1)q_1 + (p_2 - c_2)q_2 + T$$

with respect to $p_2$:

$$\frac{\partial U^I}{\partial p_2} = (p_1 - c_1) \frac{\partial q_1}{\partial p_2} + (p_2 - c_2) \frac{\partial q_2}{\partial p_2} + q_2$$

$$\frac{p_2 - c_2}{p_2} = \frac{1}{\epsilon_2} + \frac{\epsilon_{12}}{\epsilon_2} \frac{(p_1 - c_1)q_1}{p_2q_2}$$

For service 3, the entrant maximizes $U^E = (p_3 - c_3)q_3$ chooses:

$$\frac{p_3 - c_3}{p_3} = \frac{1}{\epsilon_3}$$
e. Market structure C

Complete information

Under market structure C, the regulator sets only the price for service 1 and then the firms compete on prices for service 2 and 3.

For service 3, the entrant chooses:

$$\frac{p_3 - c_3}{p_3} = \frac{1}{\epsilon_3}$$

The incumbent chooses

$$\frac{\partial U^I}{\partial p_2} = (p_1 - c_1) \frac{\partial q_1}{\partial p_2} + (p_2 - c_2) \frac{\partial q_2}{\partial p_2} + q_2 + k(p_3 - c_3) \frac{\partial q_3}{\partial p_2}$$

$$\frac{p_2 - c_2}{p_2} = \frac{1}{\epsilon_2} + \frac{\epsilon_{3,2}}{\epsilon_2} k \frac{(p_3 - c_3)q_3}{p_2 q_2} + \frac{\epsilon_{1,2}}{\epsilon_2} \frac{(p_1 - c_1)q_1}{p_2 q_2}$$

Plug the price for service 3 in the previous one:

$$\frac{p_2 - c_2}{p_2} = \frac{1}{\epsilon_2} + \frac{\epsilon_{1,2}}{\epsilon_2} \frac{(p_1 - c_1)q_1}{p_2 q_2} + \frac{\epsilon_{3,2}}{\epsilon_3 \epsilon_2} \frac{p_3 q_3}{p_2 q_2}$$

In the first stage the regulator has to define the optimal share of profit and the transfer $T$, maximizing the total welfare:

$$S(q_1) - p_1 q_1 + S(q_2) - p_2 q_2 + S(q_3) - p_3 q_3 + U^I + U^E - (1 + \lambda) T, \text{ subject to the participation constraint of the incumbent satisfied: } U^I \geq 0.$$

The objective function of the regulator can be rewritten as:

$$S(q_1) + S(q_2) + S(q_3) - [c_1 q_1 + c_2 q_2 + c_3 q_3] - \lambda T$$

Substitute for $T$, such that the participation constraint is satisfied:

$$S(q_1) + S(q_2) + S(q_3) - [c_1 q_1 + c_2 q_2 + c_3 q_3] - \lambda \left[ U^I - (p_1 - c_1) q_1 - k(p_3 - c_3) q_3 - (p_2 - c_2) q_2 \right]$$

$$S(q_1) + S(q_2) + S(q_3) - (1 + \lambda)(c_1 q_1 + c_2 q_2) - (1 + \lambda k)c_3 q_3 + \lambda(p_1 q_1 + p_2 q_2) + \lambda k p_3 q_3 - \lambda U^I$$
Under complete information:

\[ U^I = 0 \]

As we are solving backward, we plug the solutions \( q_1^*, q_2^*, q_3^*, p_2^*, p_3^* \) previously obtained in the objective function and derivate it w.r.t. \( k \).

\[
(1 + \lambda)(p_1^* - c_1) \frac{\partial q_1^*}{\partial k} + (1 + \lambda)(p_2^* - c_2) \frac{\partial q_2^*}{\partial k} + (1 + \lambda k)(p_3^* - c_3) \frac{\partial q_3^*}{\partial k} + \lambda (p_3^* - c_3)q_3^* + \\
\lambda k q_3^* \frac{\partial p_3^*}{\partial k} + \lambda q_2^* \frac{\partial p_2^*}{\partial k}
\]

**Asymmetric information**

Define \( \hat{q}_2, \hat{p}_2 \) the quantity and price in market 2 given the value of demand reported \( \hat{\theta} \).

But the incumbent maximizes its profit according to the true value of \( \theta \).

\[ U^I = [(p_1 - c_1)q_1 + k(p_3 - c_3)q_3 + T] + (p_2 - c_2)q_2 \]

Incumbent’s best response satisfies the F.O.C. as under complete information:

\[
\frac{\partial U^I}{\partial p_2} = (p_1 - c_1) \frac{\partial q_1}{\partial p_2} + (p_2 - c_2) \frac{\partial q_2}{\partial p_2} + q_2 + k(p_3 - c_3) \frac{\partial q_3}{\partial p_2}
\]

Therefore, the equilibrium prices are given by the solution of the system of the two above conditions.

In the first period, the regulator maximizes the total expected welfare with respect to \( k \) and \( T \):

\[
E(S(q_1) + S(q_2) + S(q_3) - (1 + \lambda)(c_1 q_1 + c_2 q_2) - (1 + \lambda k)c_3 q_3 + \lambda(p_1 q_1 + p_2 q_2) + \lambda kp_3 q_3 - \lambda U^I)
\]
The maximization is subject to the incentive constraint that ensures truthtelling:

\[ U^I(\theta | \theta) \geq U^I(\hat{\theta} | \theta) \]

The incumbent maximizes its utility given the second period equilibrium outcome (denoted by \( \ast \)):

\[ U^{I\ast} = \left[ (p_1 - c_1)q_1[p_2^\ast(\theta, \hat{\theta}, k), p_3^\ast(\theta, \hat{\theta}, k)] + k[p_2^\ast(\theta, \hat{\theta}, k), p_3^\ast(\theta, \hat{\theta}, k)] + T \right] + (p_2^\ast(\theta, \hat{\theta}, k) - c_2)q_2[p_1, p_2^\ast(\theta, \hat{\theta}, k), p_3^\ast(\theta, \hat{\theta}, k)] \]

Evaluate when the reported value is the true one:

\[ \left. \frac{\partial U^{I\ast}}{\partial \hat{\theta}} \right|_{\hat{\theta} = \theta} \]

The incentive constraint is:

\[ \frac{dU^I}{d\theta} = \frac{\partial U^I}{\partial \hat{\theta}} \frac{\partial \hat{\theta}}{\partial \theta} + \frac{\partial U^I}{\partial \theta} \]

Defined: \[ \frac{\partial U^I}{\partial \theta} = \dot{U}^I \]

Using the demand specification, the incentive constraint is positive.

\[ \dot{U}^I > 0 \]

Moreover, the participation constraint is binding for the lowest value of demand: \( U(\theta) = 0 \)

The Hamiltonian is \( H = W + \mu \dot{U} \)

The costate variable \( \mu \) is:

\[ \dot{\mu} = -\frac{\partial H}{\partial U} = \lambda f(\theta) \]

Using the transversality condition at \( \mu(\bar{\theta}) = 0 \) we obtain \( \mu = \lambda (F(\theta) - 1) \)
The regulator maximizes:
\[
\int_{\theta} \left[ q(S(q_1) + S(q_2) + S(q_3) - (1 + \lambda)(c_1q_1 + c_2q_2) - (1 + \lambda k)c_3 q_3 + \lambda (p_1 q_1 + p_2 q_2) + \lambda k p_3 q_3 \right] f(\theta) + \\
- \lambda (1 - F(\theta)) \hat{U} d(\theta)
\]

Considering the second period outcome \( q_1^*, q_2^*, q_3^*, p_2^*, p_3^* \)

The optimal \( k \) satisfies the F.O.C.:
\[
(1 + \lambda)(p_1 - c_1) \frac{\partial q_1^*}{\partial k} + (1 + \lambda)(p_2^* - c_2) \frac{\partial q_2^*}{\partial k} + (1 + \lambda k)(p_3^* - c_3) \frac{\partial q_3^*}{\partial k} + \lambda (p_3^* - c_3) q_3^* + \\
\lambda k q_3^* \frac{\partial p_3^*}{\partial k} + \lambda q_2^* \frac{\partial p_2^*}{\partial k} - \lambda \frac{(1 - F(\theta))}{f(\theta)} \frac{\partial \hat{U}}{\partial k} = 0
\]

As \( \frac{\partial \hat{U}}{\partial k} > 0 \) the distortion due to asymmetric information is
\[
- \lambda \frac{(1 - F(\theta))}{f(\theta)} \frac{\partial \hat{U}}{\partial k} < 0
\]

We have a downward distortion with respect to the complete information case.
A.2.2. Equilibrium outcomes

Table 1: Complete information

The tables refer to some computations of the equilibrium outcomes.
\( a_1 = 10 , a_2 = 5 , a_3 = 3 , b_1 = b_2 = b_3 = 1.9 , \ g = 0.38 , h = 1.52 , c_2 = c_3 = 1.5 , \lambda = 0.3 \)
a) \( c_1 = 1.5 \)

<p>| ( \theta )  | ( x ) | ( p_2 ) | ( p_3 ) | ( T ) | ( W ) | ( p_2 ) | ( p_3 ) | ( T ) | ( W ) | ( p_2 ) | ( p_3 ) | ( T ) | ( k ) | ( W ) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1  | 0.4  | 2.629 | 2.519 | 2.666 | 32.704 | 3.185 | 2.874 | 4.074 | 36.175 | 3.555 | 3.022 | 1.674 | 51.10% | 38.604 |
| 1.005 | 0.4  | 2.635 | 2.524 | 2.606 | 32.854 | 3.193 | 2.877 | 4.025 | 36.316 | 3.560 | 3.024 | 1.639 | 50.60% | 38.727 |
| 1.01  | 0.4  | 2.642 | 2.530 | 2.546 | 33.006 | 3.201 | 2.880 | 3.975 | 36.457 | 3.565 | 3.025 | 1.605 | 50.10% | 38.851 |
| 1.015 | 0.4  | 2.649 | 2.535 | 2.485 | 33.158 | 3.209 | 2.883 | 3.926 | 36.599 | 3.569 | 3.027 | 1.570 | 49.60% | 38.975 |
| 1.02  | 0.4  | 2.656 | 2.541 | 2.425 | 33.310 | 3.217 | 2.886 | 3.876 | 36.741 | 3.574 | 3.029 | 1.535 | 49.099 |
| 1.025 | 0.4  | 2.663 | 2.546 | 2.363 | 33.463 | 3.224 | 2.889 | 3.826 | 36.884 | 3.579 | 3.031 | 1.500 | 48.61% | 39.224 |
| 1.03  | 0.4  | 2.670 | 2.552 | 2.302 | 33.617 | 3.232 | 2.892 | 3.775 | 37.027 | 3.583 | 3.033 | 1.465 | 48.12% | 39.350 |
| 1.035 | 0.4  | 2.676 | 2.557 | 2.240 | 33.771 | 3.240 | 2.895 | 3.725 | 37.171 | 3.588 | 3.035 | 1.429 | 47.62% | 39.476 |
| 1.04  | 0.4  | 2.683 | 2.563 | 2.178 | 33.926 | 3.248 | 2.899 | 3.674 | 37.315 | 3.593 | 3.037 | 1.394 | 47.13% | 39.602 |
| 1.045 | 0.4  | 2.690 | 2.568 | 2.116 | 34.082 | 3.256 | 2.902 | 3.623 | 37.460 | 3.597 | 3.038 | 1.358 | 46.64% | 39.728 |
| 1.05  | 0.4  | 2.697 | 2.574 | 2.053 | 34.238 | 3.264 | 2.905 | 3.571 | 37.605 | 3.602 | 3.040 | 1.322 | 46.15% | 39.856 |
| 1  | 0.35  | 2.554 | 2.444 | 4.048 | 31.407 | 3.164 | 2.857 | 5.132 | 35.409 | 3.471 | 2.980 | 3.176 | 43.55% | 37.414 |
| 1.005 | 0.35  | 2.560 | 2.449 | 3.900 | 31.555 | 3.172 | 2.861 | 5.084 | 35.549 | 3.475 | 2.982 | 3.143 | 43.05% | 37.536 |
| 1.01  | 0.35  | 2.567 | 2.455 | 3.932 | 31.704 | 3.179 | 2.864 | 5.035 | 35.689 | 3.480 | 2.984 | 3.109 | 42.54% | 37.658 |
| 1.015 | 0.35  | 2.574 | 2.460 | 3.873 | 31.854 | 3.187 | 2.867 | 4.986 | 35.830 | 3.485 | 2.986 | 3.076 | 42.04% | 37.780 |</p>
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b) $c_1 = 1.9$

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Table 2 - Equilibrium outcomes – Asymmetric information

The tables refer to some computation of the equilibrium outcomes.

\[ a_1 = 30, a_2 = 8, a_3 = 7, b_1 = b_2 = b_3 = 1.856, g = 0.1, h = 1.6, c_2 = 2.2, c_3 = 1.7, \lambda = 0.3 \]

a) \( c_1 = 2.2 \)

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A.2.3. The share of participation to PSO fund

We consider only competitive segments 2 and 3.

The utility functions of the incumbent and of the entrant are:

\[ U^I = (p_2 - c_2)q_2 + k_3(p_3 - c_3)q_3 + T \]  
\[ U^E = (1 - k_3)(p_3 - c_3)q_3 \]  

3.1. \( k \) defined ex ante

Solving backward

In the second stage firms compete in prices:

\[ \frac{\partial U^I}{\partial p_2} = (p_2 - c_2) \frac{\partial q_2}{\partial p_2} + q_2 + k_3(p_3 - c_3) \frac{\partial q_3}{\partial p_2} = 0 \]  
\[ \frac{\partial U^E}{\partial p_3} = (1 - k_3) \left[ (p_3 - c_3) \frac{\partial q_3}{\partial p_3} + q_3 \right] = 0 \]

The equilibrium prices when \( k \) is decided ex ante are given by the solution of the system of the above conditions (42) and (43):

\[ p_{3,ante} = c_3 - q_{3,ante} \frac{\partial p_3}{\partial q_3} \]  
\[ p_{2,ante} = c_2 + \frac{\partial p_2}{\partial q_2} \left[ -q_{2,ante} + k_3q_{3,ante} \frac{\partial q_3}{\partial p_2} \frac{\partial p_3}{\partial q_3} \right] \]

In the first stage, the regulator sets \( k \) on the basis of the announced rule.

3.2. \( k \) defined ex post

In the second stage, the regulator observes the equilibrium outcome and determines \( k \) according to the rule defined in the first stage.
In stage 1, firms compete in prices, given the rule defined for \( k \) and the transfers \( T \).

Firm \( I \) maximizes (40.):

\[
\frac{\partial U^I}{\partial p_2} = (p_2 - c_2) \frac{\partial q_2}{\partial p_2} + q_2 + k_3(p_3 - c_3) \frac{\partial q_3}{\partial p_2} + (p_3 - c_3)q_3 \frac{\partial k_3}{\partial p_2} = 0 \tag{44.}
\]

Firm \( E \) maximizes (41.):

\[
\frac{\partial U^E}{\partial p_3} = (1-k_3)\left[ (p_3 - c_3) \frac{\partial q_3}{\partial p_3} + q_3 \right] - (p_3 - c_3)q_3 \frac{\partial k_3}{\partial p_3} = 0 \tag{45.}
\]

The equilibrium prices are:

\[
p_{3,\text{post}} = c_3 - \frac{q_{3,\text{post}}(1-k_3)}{(1-k_3) \frac{\partial q_3}{\partial p_3} - q_{3,\text{post}} \frac{\partial k_3}{\partial p_3}} \tag{46.}
\]

\[
p_{2,\text{post}} = c_2 - \frac{\partial p_2}{\partial q_2} q_{2,\text{post}} + \frac{\partial p_2}{\partial q_2} \left[ \frac{q_{3,\text{post}}(1-k_3)}{(1-k) \frac{\partial q_3}{\partial p_3} - q_{3,\text{post}} \frac{\partial k_3}{\partial p_3}} \left( k_3 \frac{\partial q_3}{\partial p_2} + q_{3,\text{post}} \frac{\partial k_3}{\partial p_2} \right) \right] \tag{47.}
\]

a) \( k \) as the ratio of quantities: \( k_i = \frac{q_i}{q_2 + q_3} \)

In this case:
\[
\frac{\partial k_3}{\partial p_2} = \frac{\frac{\partial q_3}{\partial p_2} q_2 - \frac{\partial q_2}{\partial p_2} q_3}{(q_2 + q_3)^2} > 0
\]
(48.)

\[
\frac{\partial k_3}{\partial p_3} = \frac{\frac{\partial q_3}{\partial p_3} q_2 - \frac{\partial q_2}{\partial p_3} q_3}{(q_2 + q_3)^2} < 0
\]
(49.)

**b) k as the ratio of revenues:**
\[
k_i = \frac{p_i q_i}{p_2 q_2 + p_3 q_3}
\]

In this case:

\[
\frac{\partial k_3}{\partial p_2} = p_3 \frac{p_2 q_2}{\partial p_2} \frac{\partial q_3}{\partial p_2} - q_3 \left( p_2 \frac{\partial q_2}{\partial p_2} + q_2 \right) > 0
\]

But we cannot determine the sign of the derivative with respect to \( p_3 \):

\[
\frac{\partial k_3}{\partial p_3} = \frac{p_2 q_2 \left( p_3 \frac{\partial q_3}{\partial p_3} + q_3 \right) - p_3 q_3 p_2 \frac{\partial q_2}{\partial p_3}}{(p_2 q_2 + p_3 q_3)^2}
\]
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