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To those who made me smile during these years

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Acknowledgements

1. **Chapter 2** is the reproduction of the paper *"Who is this, who enters there? Migration in Italy and its effect on fiscal sustainability and pensions"*, co-authored with Christian Hagist and Fabio Pammolli.
2. **Chapter 3** is the reproduction of the paper *"The potential costs of Longevity Risk on Public Pensions. Evidence from Italian data."*, co-authored with Luca Regis and Fabio Pammolli.
3. **Chapter 4** is the reproduction of the paper *"Intragenerational Redistribution in a Funded Pension System"*, co-authored with Giorgio Gnecco, Fabio Pammolli, and Xue Wen.

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Abstract

The ability of countries to preserve the current fiscal policy without running into solvency problems and possible default has become a great concern for many industrialized countries, which experienced significant increases in their national debt levels during the aftermath of the economic and financial crisis. Sustainable public finances and lower public debt burdens are important elements to ensure that countries are strong enough to cope with adverse macroeconomic contexts and projected implicit liabilities related to aging, i.e. pension, health care, and long-term care expenditures.

The aim of this thesis is to provide different perspectives on the study of pension systems and fiscal sustainability. In the first chapter, we estimate the impact of immigration on the sustainability of Italian public finances using the methodology of *Generational Accounting*, showing that the burden of current fiscal policy reduces as integration of the foreign-born increases. In the second chapter, we investigate how uncertainty regarding future mortality and life expectancy outcomes have affected the Italian public pension budget, revealing a consistent underestimation of actual life spans when forecasts are based on expectations. Finally, in the third chapter, we analyze in a general equilibrium framework labor market distortions and capital accumulation arising within different pension systems to highlight the importance of guaranteeing both financial viability and retirement income adequacy.

Chapter 1

Introduction

The ability of countries to preserve the current fiscal policy without running into solvency problems and possible default[1] has become a great concern for many industrialized countries who experienced significant increases in their national debt levels in the aftermath of the economic and financial crisis. Actually, government gross financial liabilities have increased, on average, from 55% of GDP in 2007 to 88% in 2014 across OECD countries (OECD (2015)).

Sustainable public finances and lower public debt burdens are important elements to ensure that countries are strong enough to cope with adverse macroeconomic contexts, such as the current low inflation and moderate GDP growth registered in most of industrialized countries. Moreover, in a longer-term perspective, solid public finances may help governments to deal with projected implicit liabilities related to aging, i.e. pension, health care, and long-term care expenditures. Actually, as shown by the long-term budgetary projections of EC (2016), aging population will translate in a significant fiscal burden threatening the financial viability of most of industrialized countries already during the next decade.[2] Furthermore, OECD (2015) estimates show that due to

[1] For a broader discussion on the concept of fiscal sustainability see EC (2016).

[2] Focusing on EU reality, EC (2016) shows that between 2013 and 2060 the projected change in public age-related expenditure (pensions, health care, long-term care and education) will be around 1.3 pp of GDP.

population aging, on average across OECD countries public pension expenditure is projected to grow from 9.5% of GDP in 2015 to 11.7% in 2050.

The aging process faced by many OECD countries is driven by declines in fertility rates and increases in life expectancy. In most industrialized countries fertility rates experienced a sharp decline by the late 1960s, falling below the replacement level around 1980, implying shrinking generations (OECD (2014b) and OECD (2015)). Conversely, increases in life expectancy of individuals, which is the main driving force behind population aging, are dated to the beginning of the 20^{th} century.[3] The increasing longevity trend has not slowed down recently, if we explore the recent period data (OECD (2016)) in OECD countries life expectancy for males aged 65 increased from 15.8 years in 2001 to 17.6 years in 2011. Likewise, for females aged 65, in the same years, life expectancy increased from 19.4 to 21 years. According to OECD (2015), the number of elderly people is projected to account for an increasing share of the total world population passing from 8% in 2015 to almost 18% by 2050, and from 16% to 27% in OECD countries. Moreover, in the OECD, the share of the population older than 75 years will be similar in 2050 to the share older than 65 years today.[4]

The observed increases in the median age of the population, which is projected to continue raising over the years ahead, have resulted in an increase in the old age dependency ratio.[5] Actually, according to OECD

[3] At the beginning of the last century, the major determinants of reductions in mortality were decreases in infant mortality and reductions in mortality at older ages caused by measures able to counteract the contraction and the spread of diseases, i.e. improvements in nutrition and public health care. Around the 1950s, mortality reduction across the age spectrum was mainly due to the developments of vaccines and antibiotics. Nowadays, chronic diseases are the major cause of morbidity and mortality (OECD (2014b)).

[4] Regarding EU and the euro area, by 2060 the proportion of people aged 0-14 is projected to remain fairly constant around 15%, while those aged 15-64 will reduce from 66% to 57%. Those aged 65 and over and those aged 80 and over will become a much larger share: by 2060, the former will rise from 18% to 28% of the population, while the latter will rise from 5% to 12% of the population (see EC (2015a)).

[5] The old age dependency ratio is the ratio of older dependents (people older than 64) to the working-age population (those aged 15-64). In 2015, the demographically oldest OECD country was Japan, with a old age dependency ratio equal to 47. Germany and Italy also had high old age dependency ratios equal to 35 and 37 respectively (OECD (2015)).

(2015), the old age dependency ratio, equal to 14 in 1950, reached the value of 28 in 2015, and it is projected to nearly double in 2075 reaching the value of 55.

Therefore, aging population poses a persistent challenge for western economies facing budgetary implications of demographic changes. According to EC (2016), a country's fiscal situation can be assessed through three main types of economic models. In particular, alongside the less frequently used macroeconomic and dynamic general equilibrium models (the latter often with overlapping-generations household structure), the quantitative confirmation of the sustainability of current policy is usually achieved via the implementation of *Generational Accounting* technique. In particular, analyses using this methodology may help in better understanding the extent to which government fiscal policies mitigate or exacerbate the economic risks facing different generations.[6]

In particular, the increasing old age dependency ratio should be seriously taken into account by governments who are financing public pensions via the Pay-as-you-go system (PAYG). In a PAYG system, active population finance pensions of same-period retirees based on the promise that they will receive a similar treatment by future workers. Due to this working mechanism, a decline in population growth may jeopardize the financial viability of the system itself since it reduces the likelihood that the promise can be maintained in the future.[7] Actually, many OECD countries providing PAYG-financed public pensions will experience financial troubles caused by the retirement of the Baby Boom generation, which is a cohort much larger than the one that followed in the workforce. Moreover, for PAYG-financed public pensions, population aging will create sustainability problems to the extent that if increases in life expectancy exceed forecasts,[8] governments will have to pay retirement benefits even more than expected exacerbating the financial burden for

[6] See Auerbach et al. (1991) and Auerbach et al. (1994).

[7] According to Samuelson (1958), Aaron (1966), and Samuelson (1975), the PAYG system, defined as the social contract between generations, is desirable only when each generations maintain positive real rates of return on contributions, which happens so long as real earnings growth and population growth remain positive.

[8] The discrepancy between actual and expected life spans and its related uncertainty is called longevity risk (see for instance IMF (2012)).

public finances.

In order to boost financial sustainability OECD (2015) and EC (2016) propose different measures, such as: increases in statutory retirement age; nominal benefits cut; increases in taxes or social security contributions on pension income, and minimum contributory periods; reductions in the valorisation of past and present pension contributions; introduction of automatic adjustment mechanisms; improvements in administrative efficiency; tighter access to early-retirement and/or increased financial incentives to work beyond the pensionable age, and higher penalties to early pension benefit withdrawal. Additionally, some countries (e.g., Italy and Sweden) have introduced the so called notional defined contribution (NDC) scheme. By maintaining a PAYG-finance, the NDC plan provides benefits that bear an actuarial relationship to individual lifetime contributions.[9] Nevertheless, due to demographic changes, the introduction of all of these measures can only partially ease PAYG system's sustainability tension.

According to EC (2015a), the most credited solution to the sustainability problem originated by PAYG system in an aging economy is the privatization of social security, i.e. the shift from the PAYG to the fully funded system (FF) where each individual builds up her own pension by contributing to a personal account.

However, population aging together with the current economic environment characterized by low returns, low growth and low interest rates, create serious problems not only for PAYG financed public pensions, but also for funded pensions (OECD (2014b)). In particular, defined benefit funded pensions that pre-commit to pay a defined pension benefit no matter on the value of assets accumulated, need to secure their continued solvency. Actually, if pension promises rely on forecasts that underestimate life expectancy of individuals, the present value of pension payments will also be underestimated and actual pension payments will be larger than expected. Conversely, defined contribution pensions, where the post-retirement consumption, given life expectancy and the interest rate, is determined only by the amount of contributions paid into the

[9] See for example Disney et al. (1999), World Bank (2005), and Holzmann et al. (2006).

fund, need to ensure income adequacy during retirement. In this case, since the future value of pension savings and the present value of pension benefits are equal by definition, there is no guarantee of whether this amount is able to ensure retirement income adequacy.

Actually, provided the heavy focus of recent pension policy actions on improving the financial viability of pension systems, any social welfare program has to guarantee an adequate standard of living in retirement (OECD (2015)).

However, the problem that every government has to face is that policy measures aimed at increasing income adequacy during retirement might add pressures on the financial sustainability of the pension system, operating thus in the opposite direction. For example, increases in benefits in a system in which there is a weak link between contributions and benefits, will affect financial balances. By the same reasoning, when public pensions are at risk of being inadequate, there will be pressure to raise benefits in order to prevent old-age poverty. According to OECD (2015), over the recent period about half of OECD countries have taken measures to improve the financial sustainability of their pension systems while about one third of the countries have improved the adequacy of retirement income for targeted groups. Countries where results of reforms are expected to be broad are those that took a combination of measures, such as increasing contributions in defined contribution schemes, increases in statutory retirement age, and introduce tighter access to early-retirement. Therefore, in light of the above discussion, in order to ensure adequate retirement benefits within a financially sustainable pension system it is important to diversify the sources to finance retirement and build a pension system that includes both a PAYG-financed component, and a funded component. The latter will include occupational as well as personal funded pension plans, normally run by private institutions. The diversification between funded and Pay-as-you-go, and between defined benefit and defined contribution enables risk mitigation as systems have different strengths and weaknesses, thus achieving a better risk-return profile of pension income (see OECD (2014b) and OECD (2015)).

Beside the introduction, the dissertation consists in three chapters that represent individual articles. Appendices are given at the end of the corresponding chapter. Each chapter of this dissertation is related to the analysis of pension systems and the assessment of fiscal sustainability.

In Chapter 2 we estimate the impact of immigration on the sustainability of the Italian public finances using the methodology of *Generational Accounting*. We take into account socio-economic differences between the main migrants communities resident in Italy and we present three possible scenarios to reflect the potential economic degree of integration of foreigners in the Italian territory. Moreover, for each scenario we propose several options for migrants concerning both the length of permanence in Italy and the possible collection of retirement benefits. Our results show that the burden of current fiscal policy reduces as integration of the foreign-born increases. If migrants children are economically perfectly integrated, the *fiscal gap* is reduced from 71.9 to -15.3 percent of GDP.

In Chapter 3 we assess, through an empirical investigation based on Italian data, how uncertainty regarding future mortality may affect public pension expenditure. Based on a representative sample of Italian pensioners from 1985 to 2011, we find a consistent underestimation of improvements seen in mortality and life expectancy when forecasts are based on expectations. The pension expenditure estimated using realized mortality rates is shown to be consistently higher than that obtained by using average forecasted scenarios, produced with well-known stochastic mortality models. The chapter highlights the importance of considering the uncertainty regarding future pension benefits, i.e. of evaluating and managing the longevity risk in public pension plans.

In Chapter 4 we analyze in a general equilibrium framework both labor market distortions and capital accumulation arising within three social security systems: a Pay-as-you-go notional defined contribution (PAYG NDC), a fully funded (FF), and a newly proposed modified version of the FF (MFF) that includes an intragenerational redistributive component able to guarantee minimum living standards to low-income retirees. While PAYG NDC depresses labor supply and physical capital accumulation, the FF is neutral on both dimensions. Conversely, the MFF is able to slightly increase physical capital accumulation (without significantly reducing labour supply incentives), and to reduce the burden of future intergenerational redistribution.

Chapter 2

Migration in Italy and its effect on fiscal sustainability and pensions

2.1 Introduction

In the aftermath of the Lehmann and Euro crisis, official sovereign debt levels have stood in the focus of the political and public discussions. In contrast, starting from the 1900s academia and think tanks have pointed out that the ability of countries to preserve their current fiscal policy without running into solvency problems and possible default should be (given the drastic ageing of many societies) the far bigger concern. The social safety net established in many countries (and therefore especially pensions, health and long-term care systems) will be under enormous fiscal pressure.

To analyze such intertemporal challenges of public coffers, classic budgetary indicators have only small informational value. However, a quantitative confirmation of the sustainability of current fiscal policy can be achieved via the implementation of *Generational Accounting*. This methodology is used to assess whether government fiscal policies mitigate or exacerbate the economic risks facing different generations (see

Auerbach et al. (1991), Auerbach et al. (1992), Auerbach et al. (1994), Kotlikoff and Raffelhüschen (1999), Raffelhüschen (1999), Hagist et al. (2009), and Bonin (2013).[1]

Notably, many authors used the *Generational Accounting* technique to argue that immigration could alleviate fiscal pressure associated with an aging population (for instance see Auerbach and Oreopoulos (2000), Bonin et al. (2000), Bonin (2013), and Rowthorn (2008)). Actually, immigration can contribute to a favorable readjustment in the age structure of the population and, by altering the size and the composition of the labor force in the receiving country, generates tax revenues to finance pensions and welfare services for the elderly (among others, see Borjas (1994), Bonin et al. (2000), and Bonin (2013)). More generally, the fiscal impact of immigration depends on the nature of the tax and benefit system in the host country (i.e. taxes paid by foreigners and welfare benefits and government services they receive), as well as on demographic and economic characteristics of the immigrants (e.g., skills, ability to find a work without displacing local workers).

In this chapter, we estimate the impact of immigration on the sustainability of the Italian welfare state using the methodology of *Generational Accounting*. In the past, Franco et al. (1992), Cardarelli and Sartor (2000), Coda Moscarola (2001), and Rizza and Tommasino (2010) applied this accounting methodology to Italy although only Coda Moscarola (2001) considered immigration.

We provide new evidences with respect to past research and the novelties can be summarized as follows. We distinguish migrants resident in Italy by their origin country in order to highlight the socio-economic differences between the major communities, namely the Albanian, the Chinese, the Moroccans, the Non-EU citizens, the Romanian and the Ukrainian. Moreover, we include three possible scenarios that reflect the potential degree of integration of migrants in Italy. Additionally, we propose several options concerning both the length of permanence on the Italian territory, and the possible collection of retirement benefits to

[1] Alongside *Generational Accounting*, it is worthwhile to mention the fiscal sustainability technique developed by Blanchard et al. (1991).

take care of the well-known phenomenon of return migration, and to take into account potential difficulties in the collection of Italian pensions after emigration.

In our analysis we find that firstly Italy is in a comparable good position from an intertemporal perspective as the *fiscal gap* in the standard scenario is only slightly positive meaning that the external debt is reduced significantly through implicit assets for which mainly the pension reform is responsible. Secondly, we find that integration is a major key for Italy to fortify its sustainable position as complete integration of the second generation of migrants reduces the *fiscal gap* again significantly. Thirdly, we find that the current regime of extreme bureaucratic hurdles for claiming pension benefits for migrants leaving Italy is giving a significant gain for Italys public coffers. As Italy is per se in a good position, this regime should be reformed to a fairer scheme.

The chapter proceeds as follows. Section 2.2 provides a general overview of the immigration phenomenon in Italy focusing on the migrants rights to access the Italian social security. Section 2.3 describes the methodology of *Generational Accounting* and the calculation of the sustainability indicators used. Section 2.4 describes the data used in the analysis, which are general government budget of a certain base-year, age- and sex-specific profiles, budgetary and population projections, and assumptions on growth and discount rates. Finally, Section 2.5 presents the results of the *Generational Accounting* using different sustainability indicators, and Section 2.6 provides some policy recommendations and concludes.

2.2 Immigration in Italy: Facts and Norms

2.2.1 Italy as a country of arrival of international migration

In the last decades, Italy has become one of the main countries of immigration in Europe. At the beginning of the 1990s, the foreign population resident in Italy amounted only to 500,000 people (ISTAT (1998)). A decade later, in 2001, the 14^{th} Census (ISTAT (2001)) registered 1.334.889 foreign individuals. That number increased by 2.5 million people in the following ten years reaching the value of 4,029,145 individuals in 2011 as reported by the 15^{th} Census (ISTAT (2011)). At the end of 2014, there were 5,014,000, foreigners residents in Italy (IDOS (2015)). According to IDOS (2015) by including Non-EU residents awaiting registration in the Italian registry offices, the total foreign presence in Italy was 5,421,000 people.

These numbers reveal that the incidence of the foreign population on the total population resident in Italy, which jumped from 2.3% in 2001 to 8.1% in 2014, is currently higher than the European average MPLS (2014e)).[2]

According to IDOS (2015) at the end of 2014, the majority of foreigners resident in Italy (more than 2.6 million) were European citizens. Slightly less than 30% of them come from an EU member state (1.5 million). An additional 20.5% (1 million people) come from Africa, and the 19.3% (969,000 individuals) come from Asia. Foreigners from the Americas were less than 7.7% (400,000 individuals), while those from Oceania together with stateless individuals were only 3000.

Top ten nationalities' share of population increased during the years even though a very high heterogeneity of origin is registered: it takes the top five citizenships to reach the 50%, and the first 16 to capture the 75% of the total number of foreign residents. In particular, the largest foreign community in Italy is the Romanian one (1,131,839 people), followed by the Albanian (490,483 people), the Moroccan (449,058 people), the Chinese (265,820 people) and the Ukrainian (226,060 people).

Foreign population growth among the years slowed the aging process of the domestic resident population. Indeed, while on average foreigners have an age of around 32 years, the Italians are 12 years older with an age of almost 45 years. In particular, in 2014, the 13.9% of the Italian population lies in the age range 0-14. The 21.3% belongs to the age class 15-35 while the 43.4% has an age between 35-64 years. Seniors, i.e. those

[2]In 2014, the incidence of the foreign population on the total population resident in European Union was 4.1%.

aged 65 and over account for the 21.4% of total Italians resident in the territory. Conversely, the majority of foreigners resident in Italy lies in the age group 15-39 years. In particular, the 19.2% are below 15 years old, the 34.3% lies in the age range 15-34, and the 43.8% belongs to the age group 35-64. Only the 2.7% of the foreigners resident in Italy is older than 65 (MPLS (2014e)).

According to ISTAT (2015), in the last ten years the foreign labor supply increased by more than one million individuals. In particular, the number of foreigners employed in the Italian labor market were 1.158.000 in 2005 rising to 2.294.000 in 2014. Nowadays, foreigners employed accounts for the 10.3% of the total employed population (IDOS (2015)). According to OECD and EU (2015) and IDOS (2015), Italy attracts a large number of labor migrants who came to fill low-skilled jobs not sought after by Italians. According to Fondazione Leone Moressa (2014), MPLS (2014e), and IDOS (2015), more than one third of foreign workers are employed in non-skilled occupations while only seven foreigners out of 100 have a skilled profession. These percentages do not change much with the length of permanence in Italy or with the length of service.[3] Moreover, foreign workers are mainly employed in jobs providing services to individuals, unskilled jobs in the industry, construction and hotel sectors. Over-qualification is a great concern. According to IDOS (2015), in 2014 there were 940.000 over-educated foreign workers, which amounted to the 41% of the total foreign occupation, a double share compared to that of Italians. As a result of their low-skilled occupations, in 2014 the net monthly pay of foreign workers was 28.5% on average lower than the one of Italians (958 Euros compared to 1,340 Euros).

Finally, it is worth mentioning that immigrant workers benefit less from welfare policies for two main reasons. First because Italy's to-

[3]According to CNEL and MPLS (2012) in 2011, there was a reduction in the employment of high skilled workers present for more than 10 years in Italy, and a reduction in the employment of those with higher education. In 2007, it was exactly the opposite. Therefore, it seems that those who stay in Italy for longer periods are the low skilled workers. On the contrary, the migration process of the high skilled workers tends to be a temporary or short-term one.

tal expenditure on social security (with the exception of contributory pensions) is less than the average expenditure of all the other EU Member States, and second because the foreign population, being on average younger than the Italian one, has a little influence of the public pension and long-term-care expenditures (EMN (2014) and IDOS (2015)). In particular, based on 2014 data, IDOS (2015) estimated that in 2013 foreigners working in Italy paid taxes for 6.1 billion Euros and social security contributions for 10.5 billion Euros, determining a total revenue for the Italian State of 16.6 billion Euros. Nevertheless, the total expenditure of the state for them was only 13.5 billion, which determined a positive balance of 3.1 billion. Moreover, in 2013 the GDP contribution produced by immigrants was equal to 123,072 billion Euros accounting for the 8.8% of Italian GDP.

2.2.2 The most relevant foreign communities in Italy

As already introduced, the main protagonists of the migratory phenomenon in Italy are the Romanian, the Albanian, the Moroccan, the Chinese, and the Ukrainian communities.

Regarding the socio-demographic characteristics of these communities (see MPLS (2014a), MPLS (2014b), MPLS (2014c), MPLS (2014d), and MPLS (2014e)) the Romanian, the Albanian, the Moroccan and the Chinese communities have a rather balanced gender structure of the population. Notwithstanding, the female share of Romanians is slightly bigger than the male share of the population, and the Albanian and Moroccan are slightly male dominated communities, but with a growing female share during the years as a result of family reunifications and births on the Italian soil. Conversely, Ukrainian community in Italy has an 80% female presence compared to a 20% male presence, highlighting a strong gender polarization. Concerning the age structure of different communities, within the Romanian, Albanian, Moroccan, and Chinese communities the young age bracket prevails with a high incidence of minors characterizing the former three communities (for Albanians and Moroccans it reach one quarter of the total). On the contrary, within the Ukrainian community the old age brackets prevail. Most of the Albanian and Moroccan presence in the Italian territory is connected to family reason while the Chinese, who are characterized by a family migratory model, apply more for residence permits connected to job reasons. Finally, the Ukrainian community with its high female share of around 50 years old tends to apply for residence permits connected to job reasons. Linked to the increase in family reunifications, during the years Italy experienced a remarkable increase of new-born immigrants in the territory.[4]

Finally, as regard the mean of permanence in the Italian territory, from ISTAT (2015) we estimate that the average length of stay is around 9.7 years for Romanians, 12.2 years for Albanians, 12 years for Moroccans and Chinese, and 10.5 years for Ukrainians.

By looking at the Italian labor market (see Fondazione Leone Moressa (2014), MPLS (2014a), MPLS (2014b), MPLS (2014c), MPLS (2014d), MPLS (2014e), and IDOS (2015)) we find that these communities are prevalently employed in non-skilled occupations. In particular, Romanian women tend to work as personal careers or domestic employees, in the hotel sector as bartender, waiter and cleaners, contrarily Romanian men are mainly employed in the construction sector as stonemasons and bricklayers. Albanians are mainly employed in the construction industry as stonemasons and in agriculture as mixed crop laborers. Moroccan men are mainly employed in the trade sector as street vendors and operators of retail sales, in construction as stonemasons, and agriculture as mixed crop laborers. The Chinese have a high proportion of self-employed workers and work especially in the trade sector, in the textile industry and the hotel sector. Ukrainian women are mainly employed as personal careers or domestic workers. Workers belonging to the Albanian and Moroccan communities receive a monthly wage that is on average just above 1000 Euros. Conversely, as a result of the specific working sector, all foreign women receive a monthly income far below their respective male part of the population (see INPS (2014)). Relevant features of each community are summarized in Table 1.

[4]According to Italian Law, individuals born in Italy to foreign parents are not Italian citizens by birth (*ius soli* does not apply).

Table 1: The most relevant foreign communities in Italy

	Residents at 1/1/2014	Female (in %)	Minors (in %)	Mean of permanence (in years)	Main sector of employment M(F)	Average annual income (in Euro)
RO	1,131,839	56.8	26.4	9.7	Construction (Domestic care, Hotel)	11,541 (7,506)
AL	490,483	48	27.5	12.2	Industry, Construction, Agriculture (Domestic care, Hotel)	14,920 (8,815)
MA	449,058	44	30.3	12	Trade, Construction, Agriculture (Domestic care)	13,700 (8,149)
CN	265,820	49	26	12	Self-employed in Trade, Textile industry, Hotel (same as M)	7,593 (6,619)
UA	226,060	79.9	8.8	10.5	Industry (Domestic care)	11,591 (8,070)

Source: INPS (2014), MPLS (2014a)-MPLS (2014e), ISTAT (2015), Strozza (2015).

Overall, in our *Generational Accounting* analysis we focus on Romanians, Albanians and Moroccans since they are very radical communities in the Italian territory with stabilization phenomena identifiable through the relevant increase of the share of long term residents (for the latter two), and family reunifications, as well as the considerable incidence of second generations. Moreover, we include the Chinese community since it is consolidating its presence in Italy especially with their propensity toward the development of autonomous enterprises and a family migration model. Finally, even though the Ukrainian community has a recent migratory history (low incidence of residence permits for long-term residents, strong gender imbalance, and scarce incidence of minors), we include it in the analysis because it is an old community implying that Ukrainians are very likely to meet the age requirement for pension eligibility.

2.2.3 Migrants' access to the Italian Social Security System

During the 1970s and 1980s, Italy experienced a series of pension reforms that, together with the aging population and the slowdown in growth[5], boosted the pension expenditure at an unsustainable level (see Fornero and Castellino (2001)). In order to provide corrections to the Italian social security system and stabilize the long-run ratio of pensions' expenditure to GDP, Italy has gone through several pension reforms over the last decades, amongst others, the gradual shift to the Notional defined contribution (NDC) scheme,[6] the revision of the NDC transformation coefficients (which account for the increase of life expectancy), the introduction of an automatic link between life expectancy and pensionable age, the reduction in benefits for future retirees, and the increase in the age at which people can first claim pensions. These measures helped to contain the explosion of the pension spending in the long run. Actually, according to OECD (2015), the Italian public spending on pensions is forecasted to slightly reduce until 2060.

With the entry into force of the last pension reform, the Fornero reform (Law 213 of December 22^{nd} 2011), all contributions paid after January 1^{st} 2012 are calculated according to the NDC system. Moreover, getting access to the old age pension requires an age of 66 years[7] in addition to a contribution requirement of at least 20 years. Conversely, being entitled to the early retirement benefit now demand a contribution re-

[5] Aging population implies a lengthening in pensions' payment period, while the slowdown in growth implies a reduction in the amount of contributions used to finance retirees' pensions.

[6] NDC pensions were introduced in Italy after 1/1/1996. In particular, workers with more than 18 years of contribution at 12/31/1995 would have received a defined benefit (DB) pension; conversely, individuals hired after 1/1/1996 would have obtained the NDC pension. Moreover, workers with less than 18 years of contribution at 12/31/1995 would have obtained a mixed pension, i.e. a benefit computed according to both schemes DB and NDC.

[7] Starting from January 1^{st} 2012, the age requirement for the old age pension was set at 66 years for all male employees and self-employed and for women working in the public sector. In 2012, employed women in the private sector could retire at 62 while self-employed women were allowed to retire at 63 years and six months. The age requirement is raised up to 66 years starting from 2018, and will continue to raise in line with life expectancy.

quirement of 41 years and ten months for women and 42 years and ten months for men. Such requirement gradually raise in line with life expectancy.[8]

As regards the social security provisions for immigrant workers living in Italy, the Italian Law establishes that they are the same that apply to Italian workers. However, in the case of repatriation,[9] the equality of treatment between Italians and migrants is guaranteed only when the foreign born accrue autonomously the right to receive a pension, or when they become eligible for retirement by means of the aggregation of social security contributions paid in Italy and in the origin country.[10]

Indeed, aiming to strengthen the cooperation between national social security authorities, Italy adopted the Regulation no. 883/2004 and signed several bilateral agreements with third country nationals admitting the aggregation of social insurance periods spent in contracting States to reach the pension eligibility requirements established by national laws. Under this system, the amount of the pension is determined by each country according to its own system of calculation and in proportion to the insurance periods completed under the national legislation (the pro-rata system). The Regulation no. 883/2004 applies to all EU Country members[11] and to Switzerland, Norway, Liechtenstein and Iceland, and is directly applicable by contracting member states. Conversely, bilateral

[8]As regards the age requirement, the reform provided a reduction in the pension entitlements for those workers who choose to collect their early pension before the age of 62.

[9]In the case of repatriation, the requisites to have access to old-age pensions differ with respect to the contributory and non-contributory schemes (Circolare INPS March 14, 2011 n.35). If migrants are entitled to NDC, they can collect their pension when they reach the age requirement provided by law (they do not have to meet the contribution requirement). Conversely, they can receive the DB or Mixed pension only if they satisfy both age and contribution requirements.

[10]Individuals can aggregate contributions of the following types: mandatory, imputed (military service, sickness, maternity, ordinary redundancy fund, unemployment, tuberculosis, mobility), voluntary, and others (redemption of the period of university education, omitted contributions and contributions for an activity carried out in foreign countries with no agreement in force).

[11]Countries considered are Austria, Belgium, Bulgaria, Cyprus, Croatia, Denmark, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Leetonia, Lithuania, Luxemburg, Malta, Netherlands, Poland, Portugal, United Kingdom, Czech Republic, Romania, Slovakia, Slovenia, Spain, Sweden, and Hungary.

agreements with third country nationals must be ratified by the Italian parliament in order to entry into force. So far, Italy has signed and ratified bilateral agreements with: Argentina, Brazil, Canada, Cape Verde, Israel, Jersey, Principality of Monaco, Republic of San Marino, United States, Tunisia, Uruguay, Vatican Holy See, Venezuela and the Republics of the former Yugoslavia, namely Bosnia-Herzegovina, Macedonia, Serbia, Montenegro, Kosovo.

Obviously, a "basic protection" of the rights of third country nationals applies even if no bilateral agreement on social security is in force. In that case, workers who return to their home country maintain the social security rights already accrued, but they will only be able to enjoy them after reaching the eligibility requirements provided by the Italian law. Moreover, if the contribution requisite is not satisfied, foreign citizens who meet the age requirement may apply for the quota of their pension, which correspond to their reduced contributions (see EMN (2014) and INPS (2015)).

Regarding the major communities in Italy, Romanians have less stringent rules for exportability of social security benefits than Albanians, Moroccans, Chinese and Ukrainians. Actually, Romanians eligible for retirement may demand the payment of benefits abroad. The only exception regards the payment abroad of non-contributory pensions. Social allowances (*pensione sociale* and *assegno sociale*) which are based on age and income rather than contributions paid, and other welfare benefits (such as pensions and allowances in favor of the blind, deaf and disabled people) cannot be exported. Conversely, Albanians, Moroccans, Chinese and Ukrainians are not covered by a bilateral agreement on pensions and for this reason they have a less favorable treatment than Romanians. For them pensions, with the exception of welfare benefits (i.e. non-contributory benefits) and accident insurance benefits are exportable abroad, while benefits related to illness, maternity, unemployment and ordinary redundancy are not.

As highlighted by EMN (2014), foreign workers, who usually have a fragmented career and are often forced to return home prematurely, due to loss of their jobs or the non-renewal of their residence permit, may be not aware of the possibility of applying for a pro-rata while living abroad, and, even those who do know about this possibility, may not know how to apply for it. Actually, according to INPS (2015), at the end of 2014, the 21% of foreign born workers aged 66 and over did not receive any retirement benefit for the contributions paid to the Italian social security (they were 198.430 out of 927.448 individuals).

As argued by Holzmann et al. (2005) when the portability of social security contributions is not properly guaranteed, the enrollment in the informal sector becomes more attractive for migrants. Indeed, migrants who wishes to return to their home country after a period spent working abroad and who are not sure of being able to keep their benefits face high incentives to avoid paying social security contributions during the stay in the host country. Consequently, migrant workers will seek to participate in the informal sector of the host country instead of the formal sector. Enhancing portability of long-term social security benefits may therefore be a useful tool for migrant-receiving countries to encourage migrants to participate in the formal sector and discourage irregular migration.

2.3 The Methodology of *Generational Accounting*

To measure the sustainability of a country's public sector we use the method of *Generational Accounting* developed by Auerbach et al. (1991), Auerbach et al. (1992), and Auerbach et al. (1994). In contrast to traditional budget indicators which are based on annual cash flow budgets, *Generational Accounting* is founded on the intertemporal budget constraint and therefore the long-term implications of a current policy can be computed.[12] The intertemporal budget constraint of the public sector, expressed in present value terms of a base-year b is:

[12] The further description of the methodology of *Generational Accounting* is mainly based on Raffelhüschen (1999), Hagist et al. (2011), Bonin (2013). For an analytical derivation of the intertemporal budget constraint see Benz and Fetzer (2006) or Fetzer (2006). Hagist (2008) gives an overview about the empirical studies with *Generational Accounting* along with a discussion concerning critical points in theoretical as well as empirical terms.

$$B_b = \sum_{k=b}^{b-D} N_{b,k} + \sum_{k=b+1}^{\infty} N_{b,k} \tag{2.1}$$

D denotes agents' maximum age and $N_{b,k}$ the present value of year b's net tax payments, i.e., taxes paid net of transfers received, made by all members of a generation born in year k over the remaining life cycle. Then, the first right-hand term of Eq. 2.1 represents the aggregate net taxes of all generations alive in the base-year b. The second term aggregates the net tax payments made by future generations born in year $b+1$ or later. Together this is equal to the left-hand side of Eq. 2.1, B_b, which stands for the net debt in year b. That means if the sum of all living generations' net taxes, $\sum_{k=b}^{b-D} N_{b,k}$, is negative (i.e. if they receive a net transfer) and the net debt, B_b, positive, the sum of future generations' net taxes has to be positive to balance the government's intertemporal budget i.e. in a long-term perspective net transfers received by living generations plus the net debt of the base-year have to be financed by net taxes paid by future generations.

To calculate generations' aggregated life cycle net tax payments, the net payment terms in Eq. 2.1 are decomposed into:

$$N_{b,k} = \sum_{s=\max\{b,k\}}^{k+D} T_{s,k} P_{s,k} (1+r)^{b-s} \tag{2.2}$$

In equation 2.2, $T_{s,k}$ denotes the average net tax paid in year s by a representative member of the generation born in year k, whereas $P_{s,k}$ stands for the number of members of a generation born in year k who survives until year s. To compute the remaining lifetime net payments of living generations, the future demographic structure is specified conducting long-term population forecasts.

Typically, *Generational Accounts* disaggregate Eq. 2.2 even further. To incorporate gender-specific differences in average tax payments and transfer receipts by age, separate aggregation of the average net taxes paid by male and female cohort members is required. The products aggregated in Eq. 2.2 represent the net taxes paid by all members of generation k in year s. For generations born prior to the base-year the summation starts from year b, while for future born cohorts, the summation starts in year $k > b$. Irrespective of the year of birth, all payments are discounted back to the base-year b by application of a real interest rate r.

The age-specific net tax payment in year s of agents born in year k can be decomposed as:

$$T_{s,k} = \sum_i h_{s,k,i} \tag{2.3}$$

$h_{s,k,i}$ stands for the average tax or transfer of type i paid or received in year s by agents born in year k, thus of age $s-k$. In Eq. 2.3, $h > 0$ indicates a tax payment, whereas $h < 0$ defines a transfer.

Applying the method of *Generational Accounting* it is conventionally assumed that initial fiscal policy and economic behavior are constant over time. Under this condition it is possible to project future average tax payments and transfer receipts per capita from the base-year age profile of payments according to:

$$h_{s,k,i} = h_{b,b-(s-k),i}\,(1+g)^{s-b} \tag{2.4}$$

where g represents the annual rate of productivity growth. Eq. 2.4 assigns to each agent of age s-k in year k the tax and transfer payment observed for agents of the same age in base-year b, uprated for gains in productivity. The base-year cross section of age-specific tax and transfer payments per capita is generally determined in two steps. First, the relative position of age cohorts between themselves in the tax and transfer system is estimated from micro-data profiles. In a second step the relative age profiles are re-evaluated proportionally to fit the expenditure and tax revenues of the base-year.

For living and future generations, division of the aggregate remaining lifetime net tax payments by the number of cohort members alive in year

s defines the cohort's *Generational Account* in year s:

$$GA_{s,k} = \frac{N_{s,k}}{P_{s,k}} \tag{2.5}$$

Generational Accounts are constructed in a purely forward-looking manner, only the taxes paid and the transfers received in or after the base-year are considered. As a consequence, *Generational Accounts* cannot be compared across living generations because they incorporate effects of differential lifetime. One may compare, however, the *Generational Accounts* of base-year and future born agents, who are observed over their entire life cycle.

To illustrate the fiscal burden of current fiscal policy we use four sustainability indicators:[13] The starting point for the first indicators are the *intertemporal public liabilities* which can be computed by the assumption that the intertemporal budget constraint of the public sector in Eq. 2.1 is violated:

$$IPL_b = B_b - \sum_{k=b-D}^{\infty} N_{b,k} \tag{2.6}$$

The amount of *intertemporal public liabilities* measures aggregate unfunded claims on future budgets, assuming that the present policy will hold for the future. The first of our used sustainability indicator, the *fiscal gap* (FG_b), can be derived if the *intertemporal public liabilities* are set in relation to the GDP of the base-year (GDP_b). This indicator is akin to the debt quota well known since the Maastricht treaty but it addresses the total debt, i.e. the debt which will occur in the future added to the debt inherited from the past:

$$FG_b = \frac{IPL_b}{GDP_b} \tag{2.7}$$

[13] For a discussion of measuring fiscal sustainability and the development of sustainability indicators, see Raffelhüschen (1999) and Benz and Fetzer (2006).

How the policy adjustment required to redeem *intertemporal public liabilities* will affect generations' fiscal burdens is uncertain. For illustrative purposes, *Generational Accounting* typically assigns the entire adjustment to future generations which is equivalent to $k > b$. All tax payments made by members of future born cohorts are adjusted proportionally with the help of a uniform scaling factor θ. The factor θ is set to ensure balance of the intertemporal public budget defined in Eq. 2.1:

$$h_{s,k,i} = \theta\, h_{b,b-(s-k),i}\, (1+g)^{s-b} \tag{2.8}$$

for and instead of Eq. 2.4. Computing the average age-specific net taxes paid by representative future born agents, the burden for future generations can be illustrated as an absolute difference between the *Generational Account* of the base-year agent and the *Generational Account* of the one year after base-year born agent. This is our second sustainability indicator, the *future generations' burden*:

$$FGB = GA_{b,b} - GA^{\theta}_{b,b+1} \tag{2.9}$$

The third indicator that illustrates the burden of current fiscal policy is the *revenue gap*. In this case the scaling factor $\theta = \theta_{rev}$ reflects the enhancement of age-specific revenues in per cent for all generations which is necessary to close the intertemporal public budget constraint. It can also be interpreted as the ratio of the *intertemporal public liabilities* to the present value of all age-specific revenues of the fiscal system:

$$\theta_{rev} = \frac{IPL_b}{\sum_{s=b}^{\infty} Rev_s \frac{1}{(1+r)^{(s-b)}}} \tag{2.10}$$

with Rev_s referring to the sum of revenues in year s by all living generations in year s. Analogous to the *revenue gap*, we compute also the so-called *transfer gap*. In this case the scaling factor $\theta = \theta_{trf}$ reflects the necessary decrement of age-specific public transfers (*Trf*) like health benefits in per cent for all generations that is necessary to close the intertemporal public budget constraint. Constructing the *revenue* and *transfer gap*, we implicitly assume that the government is able to enforce an immediate adjustment of all taxes and contributions or transfers respectively.

All used indicators are defined using an infinite time horizon. In the practical calculation, all relevant variables like population or cohorts tax payments are projected for 300 years from the base-year on. Afterwards a geometric series is used to determine the remaining net tax payments. The choice of 300 periods is nearly completely arbitrary and just reflects a good approximation point for our analysis.

2.4 Data and Assumptions

2.4.1 Budgetary data

Aggregates for revenues are taken from ISTAT (2014b) and aggregate for expenditure are taken from EC (2015a) and Eurostat (2016a). They are based on detailed statistics about tax and social contribution receipts as well as the government expenditure by function according to the classification of the functions of government (COFOG), the harmonized classification on the international level.

As shown in Table 2, revenues include taxes on labor and capital incomes, taxes on consumption, property taxes and social insurance contributions. Expenditures refer to government consumption, and thus to general public services, defense, public order and safety, economic affairs and environmental protection, housing and cultural activities. Moreover, health expenditure here is divided into four subcategories, namely the in- and outpatient sector, pharmaceutical expenses, and miscellaneous expenditure. Education is parted in four main categories, namely pre-primary and primary education, secondary education, post-secondary education and higher education. Finally, the social protection expenditure is mostly related to old age pensions, sickness and invalidity pensions, survivor pensions, and family and unemployment allowances. In total, revenues were 781.6 billion Euros while public expenditure al-together made 829.1 billion Euros. This results in a deficit of 47.5 billion Euros. This deficit minus interest payments on the Italian gross financial liabilities made up the primary surplus with 30.5 billion Euros.

Table 2: Budget of the Italian Government in 2013

Public Expenses (billion Euros)		**Public Revenues** (billion Euros)	
Government Consumption	243.5	Personal Income Tax (IRPEF)	176.3
Long-term Care	29.1	Corporate Income Tax (IRES)	38.8
Inpatient Medical Services	49.7	Other direct taxes	26.9
Outpatient Medical Services	36.3	Value Added Tax (VAT)	93.8
Pharmaceutical Services	10	Enterprise Tax (IRAP)	32
Other Health-related Expenses	2.8	Municipal Property Tax (IMU)	19.2
Pre-school and Primary Education	21.8	Excise taxes and similar taxes	57.8
Secondary Education	26.5	Other indirect taxes	36.8
Post-secondary Education	1.1	Employers social contributions	148.4
Higher Education	5.4	Employees social contributions	37.2
Other Education-related Expenses	5.1	Self-employed social contributions	28.8
Old Age Public Pension	204	Unemployed social contributions	0.8
Survivors Public Pension	40.2	Capital Income Tax	4.1
Sickness an Invalidity Public Pension	35.7	Other revenues	80.7
Family allowances	21.7		
Unemployment allowances	18.2	*Deficit*	*47.5*
Interest Payment	77.9		
Primary Surplus	*30.5*		

Source: ISTAT (2014b), EC (2015a), Eurostat (2016a).

2.4.2 Micro profiles

The age- and sex-specific micro profiles are necessary to define the intertemporal budget constraint of the public sector. These profiles are used to distribute the different aggregates of public revenues and expenditures on the cohorts which live in the base-year and hence to determine the future public revenues and expenditures. The micro profiles stem from various sources. Non age-specific revenues and expenditures like defense are distributed with a flat per capita profile. Public pension profiles are taken from ISTAT (2014c); we allocate separately old-age pensions, survivors pensions, disability allowances (invalidity pensions - *pensioni di invalidità*, and pensions for accidents at work and occupational diseases - *pensioni INAIL*), and non-contributory pensions transfers (disability pensions - *pensioni di invalidità civile*, social pensions – *pensioni e assegni sociali*, and war pensions). Health expenditure profiles stem from the Survey on the Health Conditions and Access to Health Services, which is conducted by ISTAT (2014a); we separately consider pharmaceutical services, in- and outpatient medical services, and general medical services. Moreover, the profile for long-term care stems from the EC (2015a). Education profiles are taken from Eurostat (2016b); we divide between four main categories, namely pre-primary and primary education, secondary education, post-secondary education, and higher education. Most of the other profiles are based on the Household Survey of Income and Wealth (SHIW), which is conducted by Banca dItalia (2014). In particular, on the revenue side we allocate taxes to different age groups by considering the age and sex distribution of the relevant tax bases taken from SHIW. We allocate social security contributions using labor income, value added taxes using consumption,[14] taxes on financial income using the age-sex distribution of financial assets, and taxes on real estate income using the value of the properties owned. Italian micro profiles are illustrated in Figure 3 in Appendix A.1.

To specifically account for the peculiar characteristics of the foreign population residents in Italy, we construct micro profiles for the five major communities of migrants, namely for Albanians, Chinese, Moroccans, Non-EU citizens, Romanians and Ukrainians. Migrants' profiles are primary based on information contained in MPLS (2014a), MPLS (2014b), MPLS (2014c), MPLS (2014d), MPLS (2014e), INPS (2014), and Strozza (2015) and are built by means of a rescaling process of the Italian micro profiles. A particular attention is devoted to the construction of migrants' pension profiles where, in the rescaling process, we include the community-specific mean of permanence in the Italian territory (see Section 2.2.2). Health, long-term care, and education expenditure profiles make an exception since are assumed equal to those of Italians. Original data used to build micro profiles for migrants are reported in Figure 4 in Appendix A.1.

[14] In the SHIW survey, consumption is reported at the household level. Therefore, we construct the age profile for the VAT by allocating consumption to each member on the basis of personal income.

2.4.3 Budgetary and Demographic projections

Budgetary projections related to health, education, and long-term care expenditures are taken from official forecasts of the EC (2015a). Differently, budgetary projections related to social security stem from an own model that incorporates EC (2015a) estimates to take into account the existence of different pension schemes (defined benefit, mixed, and notional defined contribution plans).

Regarding demographic projections, for both Italians and foreigners we use the data for the base-year and the assumptions for the developments of fertility, mortality and net migration provided by Eurostat (2013). We then calculate an own demographic projection for 300 periods (see Section 2.3) using an application of the cohort-component method provided by Bonin (2013).

Projections of annual migrants inflows in Italy are taken from Eurostat (2013), and in order to preserve the different characteristics of the foreign communities under review, we fix their relative share (over the total foreign population) at the one observed in the base-year (see MPLS (2014e)). Furthermore, in annual migrants inflow projections, we grouped the major EU communities, namely Bulgaria, Poland and Germany, and we treat them as Italians. Moreover, in order to make clear the effect on the Italian social security system, we assume that Italians and foreigners share the same fertility and mortality rates.[15]

Finally, we set a growth rate of 1.5% per annum and a (real) discount rate of 3% per annum to predict the future revenues and expenditures of the public sector and to analyze the sustainability of this system. These assumptions are taken from EC (2015b) as the long-term equilibrium rates (which every economy reaches until 2060 the latest).

[15] We address to future research whether and how differences in fertility and mortality rates between Italians and foreigners affect our calculations.

2.5 The role of immigration in the sustainability of Italy's fiscal system

In our sustainability analysis, we include three possible scenarios that reflect the potential degree of integration of migrants in Italy. In the first scenario, we treat all migrants and newcomer migrants as foreigners. In the second scenario, we make a step forward in the integration process of migrants: we treat Romanian children younger than 11 in the base-year as natives, while, due to socio-economic differences with Italians, we continue to treat the rest of Romanian population, newcomer Romanians, and all other migrants as foreigners.[16] Finally, in the third scenario we assume a higher level of integration. In particular, we treat all migrants children younger than 11 in the base-year as Italians, and, due to socio-economic differences with natives, we consider the rest of migrants and the newcomer migrants as foreigners.

For each of the aforementioned scenarios we propose several options concerning both the collection of retirement benefits (see Section 2.2.3 for discussion) and the length of permanence on the Italian territory (the phenomenon where individuals decide autonomously to return to their country of origin after a period of work in the receiving country is a high relevant feature in practice[17]). In particular, we propose two extreme options: in the first one all foreigners spend their whole life in Italy and collect the pension, while in the second one all migrants leave Italy at the age of 65 without claiming any retirement benefit.[18] Moreover, we introduce seven intermediate options where all foreigners leave Italy after turning 65, but only selected communities obtain the pension benefit. In particular, for these intermediate options, we first suppose that

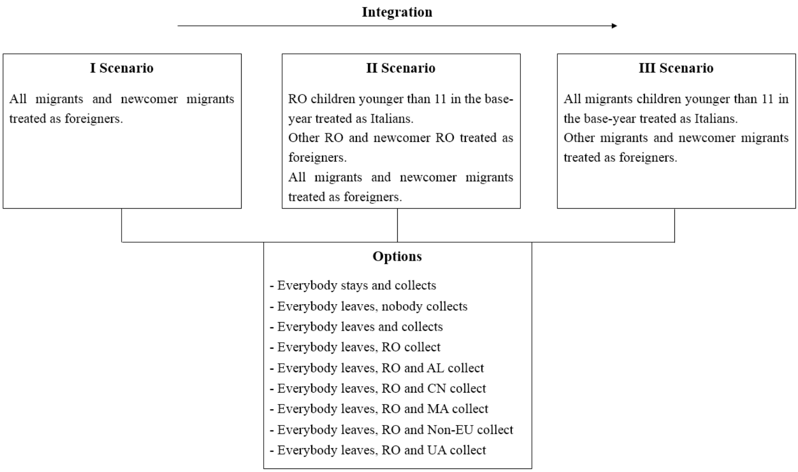
[16]In the second scenario, we decide to treat only Romanians children as Italians. By belonging to UE, Romanians have freedom of movements, and unlike all the other considered communities, they do not need a residence permit for staying in the Italian territory.

[17]See for instance Borjas and Bratsberg (1994), Dustmann (1996), and Dustmann (2003).

[18]The assumption of return migration at age 65 is reasonable and not relevant in determining how much migrants contribute to the Italian social security. Actually, in the construction of migrants' pension profiles we include the community-specific mean of permanence in the Italian territory that we use as a proxy for the length of contribution to the Italian social security system.

all foreigners claim the pension, and then we assume that only Romanians collect. Afterwards, we assume that only Romanians and one other community among the Albanian, the Chinese, the Moroccan, the Non-EU citizens, and the Ukrainian obtain the pension. Figure 1 summarizes the proposed scenarios and options.

Figure 1: Scenarios and options



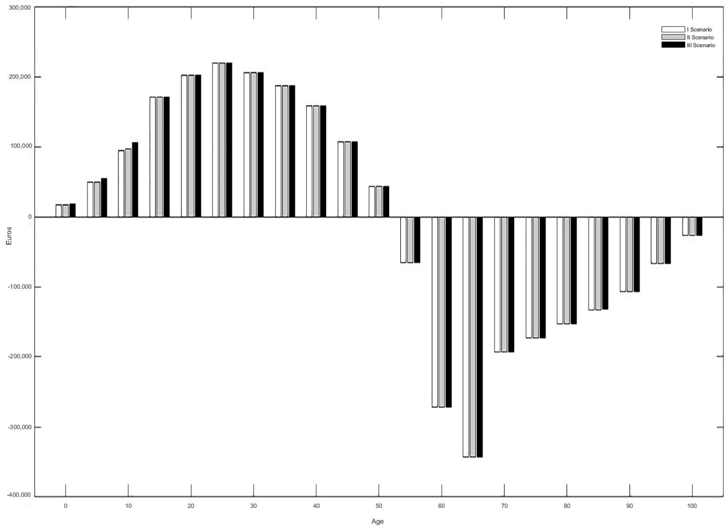
Source: Own hypotheses

2.5.1 *Generational Accounts*

Using the age- and sex-specific micro profiles, and the budgetary and population projections, we calculate *Generational Accounts* of currently living cohorts under the assumption that current policies remain unchanged. In particular, Figure 2 presents the *Generational Accounts* of the total population resident in Italy (i.e. Italians plus migrants) when, under different scenarios, everybody collects their pension and spends their retirement period in Italy.

The sinus-shaped pattern is very common in OECD countries with strong Pay-as-you-go systems. Irrespective on the scenario, current new-

Figure 2: *Generational Accounts* of Italy 2013: Everybody stays and collects the pension in different scenarios



Source: Own calculation

born and individuals aged between 1 and 53 years are net payers to the government while generations from 54 years and over are net receivers.[19]

As expected, strong differences among proposed scenarios exist only for children aged 0 to 10 years, i.e. those born between 2003 and 2013.

With the low integration assumed in the first scenario, current newborn are expected to pay to the government net taxes for 16,400 Euros. Moving integration forward, the same individuals have to pay higher net taxes equal to 16,530 Euros in the second scenario and to 18,139 Euros in the third.[20] *Generational Accounts* are at a maximum of 221,500 Eu-

[19]Compared to other analyses from other countries, the Italian newborn is a net-payer (see for example Hagist (2008) who shows that in Austria, France, Germany, Switzerland, the UK and the US, newborns are always net-receivers. This is entirely due to the pension-reform in Italy as if we assume that the old-pension system would be in place, Italian newborns also become net receivers (negative generational accounts).

[20]The increasing pattern in net taxes paid when passing from the first to the third sce-

ros in scenarios I and II, and of 221,510 Euros in scenario III, paid by a 27 years old individual. This means that a 27 years old person pays 221,500 Euros more in taxes and contributions over his/her remaining life-cycle than he/she will receive in transfers and subsidies from the Italian general government. The generation of 54 years is the first one which receives more than he/she pays in taxes over his/her remaining life cycle. However, one should keep in mind that *Generational Accounting* is strict forward looking so living generations' accounts are not comparable. Finally, we observe that the major receiver is the generation of 67 years olds: 347,910 Euros in the first scenario, and 347,900 Euros in the second and third scenario.

2.5.2 The *Fiscal gap* and other sustainability indicators

Our first sustainability indicator is the *fiscal gap* as defined in Eq. 2.7. It measures the sum of the *Generational Accounts* for living and future generations, weighted with their (expected) cohort size, set in relation to base-year's GDP. By looking at Table 3, in the reasonable hypothesis where all foreigners decide to migrate back in their origin country, but only Romanians collect the pension from abroad, the *fiscal gap* for the whole Italian public sector is -80.5% in the first scenario, -81.7% in the second, and -89.9% in the third.[21] This means that the observed migrants' tendency of leaving Italy without claiming the retirement benefit translates in a huge gain for Italy in terms of higher fiscal sustainability. Notably, the *fiscal gap* reduces as we move integration forward, i.e. if we go from the first to the third scenario.

Our second indicator is the *future generations' burden*. To calculate this indicator, the *intertemporal public liabilities* and the number of people in future generations are set in proportion to each other (Eq. 2.9). This indicator implies that the entire adjustment is borne by future generations. The burden for future generations can be illustrated as an ab-

nario is particularly relevant for all the cohorts born between 2003 and 2013. Migrant children pay more in net taxes when are treated as Italians than as migrants.

[21] The *fiscal gap* is positive if a government is in debt i.e. if the demographic development puts a burden on public coffers. Hence, a negative algebraic sign imputes a net wealth over the long-term of the country's fiscal policy.

solute difference between the generational account of the base-year and the generational account of the one year after base-year born agent. In the same hypothesis where migrants return to their home countries, but where only Romanians collect the benefit from abroad, the public sector assigns a future generations' burden of -33.8 base-year GDP in the first scenario, -33.7 in the second, and -35.0 in the third. One has to keep in mind that changing the assumptions from scenario I to scenario II (and then III subsequently) alters the socio-demographic structure of the population. This is why comparing the above example, the *fiscal gap* shows an improvement (from -80.5 (scenario I, everybody leaves, RO collect) to -81.7) which is also reflected in the revenue and transfer gap (from -2.800 (3.165) percent in scenario I to -2.801 (3.173)). Therefore, at a first glance it may be counterintuitive that the indicator *future generations' burden* is decreasing from a (positive) value of 33,800 Euro (scenario I) to 33,700 Euro (scenario II). However, economically this just reflects that the "gain" in the *fiscal gap* from changing the socio-economic structure of children with a migration background is stemming more from higher net taxes paid by future generations than by living ones. To put it in other words, the share of the present value of the sum of higher net taxes (i.e. a better *fiscal gap*) of future generations is higher than that of the living ones in scenario II and III. Therefore, the *future generations burden* (or in this case better future generations' benefit) is higher (respectively the benefit is reduced) between the two scenarios with a higher degree of integration (from scenario I to III).

The future born agent has to carry a lower burden compared to the corresponding living generation. This is also reflected in our last two sustainability indicators, the *revenue* and *transfer gap*. In the same situation where all foreigners decide to return to their origin country, but where only Romanians collect the pension from abroad, Italys government would have to cut all taxes by 2.8% in scenario I and II, and by 2.9% in scenario III. Alternatively, Italys government could increase all transfers by 3.2% in the first two scenarios, and by 3.3% in the third.

Table 3: Sustainability Indicators

	Fiscal gap (% GDP 2013)	***Future generation's burden*** (% GDP 2013)	***Revenue gap*** (%)	***Transfer gap*** (%)
I Scenario				
E stays and collects	71.9	17.1	2.5	-2.6
E leaves, nobody collects[22]	-98.5	-51.9	-3.4	3.9
E leaves and collects	-18.2	-12.0	-0.6	0.7
E leaves, RO collect	-80.5	-33.8	-2.8	3.2
E leaves, RO & AL collect	-73.2	-31.3	-2.5	2.9
E leaves, RO & CN collect	-76.0	-32.3	-2.6	3.0
E leaves, RO & MA collect	-73.0	-31.2	-2.5	2.9
E leaves, RO & Non-EU collect	-41.0	-20.0	-1.4	1.6
E leaves, RO & UA collect	-76.8	-32.5	-2.7	3.0
II Scenario				
E stays and collects	57.5	12.9	1.9	-2.1
E leaves, nobody collects	-92.9	-46.9	-3.2	3.6
E leaves and collects	-19.4	-12.3	-0.7	0.7
E leaves, RO collect	-81.7	-33.7	-2.8	3.2
E leaves, RO & AL collect	-74.4	-31.1	-2.6	2.9
E leaves, RO & CN collect	-77.2	-32.2	-2.7	3.0
E leaves, RO & MA collect	-74.2	-31.1	-2.5	2.9
E leaves, RO & Non-EU collect	-42.2	-20.1	-1.4	1.6
E leaves, RO & UA collect	-78.0	-32.4	-2.7	3.0
III Scenario				
E stays and collects	-15.3	-7.5	-0.5	0.5
E leaves, nobody collects	-101.1	-39.1	-3.3	3.8
E leaves and collects	-51.4	-20.8	-1.7	1.9
E leaves, RO collect	-89.9	-35.0	-2.9	3.3
E leaves, RO & AL collect	-85.5	-33.4	-2.8	3.2
E leaves, RO & CN collect	-87.1	-34.0	-2.9	3.2
E leaves, RO & MA collect	-85.3	-33.3	-2.8	3.2
E leaves, RO & Non-EU collect	-65.2	-25.9	-2.1	2.4
E leaves, RO & UA collect	-87.5	-34.1	-2.9	3.2

Source: Own calculation based on EC (2015a), Eurostat (2013) MS, $g = 1.5\%$, $r = 3\%$

[22] The counter-intuitive result that the indicator *fiscal gap* shows a superior situation in scenario I ''Everybody leaves, nobody collect" than in II ''Everybody leaves, nobody collects" (-98.5 to -92.9) can be explained as follows. As the Romanian community in Italy is by far the largest in numbers, the populations differ quite significantly in size between both scenarios. Therefore, the lower *fiscal gap* is due to a smaller population size, especially if all potential pensioners leave Italy. However, for the indicator *future generations burden* the absolute value of the sustainability is again relevant as in this thought experiment the gap is just closed by future generations (Eq. 2.9). Depending on the sizes of these generations, the outcome could differ from the *revenue* and *transfer gap* (basically depending on the fertility rate). Therefore, it still holds that the situation is superior if in Italy born foreigners are treated as Italians.

2.6 Discussion of results and policy recommendations

In this chapter, we use the methodology of *Generational Accounting* to study the sustainability and the intergenerational effects of the Italian current fiscal policy. Motivated by the important role that Italy is playing nowadays as a country of arrival of international migration, we provide new evidences on the impact of immigration on the sustainability of the Italian welfare state. A particular attention is devoted to the distinction of migrants resident in Italy with respect to their origin country to highlight their specific socio-economic characteristics and then, to estimate the influence that each group has on the sustainability of public finances. Hypothesis regarding the level of integration of foreigners, their length of permanence on the Italian territory, and the possible collection of retirement benefits in case of repatriation are made to create a more realistic analysis that may be used to provide improvements to the Italian social security system.

In our analysis, we find that, given the assumptions of the Ageing Working Group of the European Commission (EC (2015a)), Italy is from the perspective of fiscal sustainability in a formidable position, as the fiscal gap in the standard scenario is only slightly positive. As Moog et al. (2015) are showing Italy is even the European frontrunner given the assumptions used. Nonetheless, if we loosen the assumptions especially regarding health and long-term care expenditures, Italy joins the ranks of European countries with a *fiscal gap* comparable to others on relative good terms.

Moreover, we provide new empirical evidences indicating that enhancing integration of immigrants within economic realm allows greater fiscal sustainability for the Italian economy. Actually, a complete economic integration of the second generation of foreigners, that we simulate in the third scenario of our analysis by treating children with a migration background up to ten years old and future migrants newborns as economically equal to Italian children, results in a massive improvement of sustainability conditions corresponding to nearly 87 per cent of GDP.

Furthermore, our examination shows that the current regime of extreme hurdles in collecting accrued pension benefits for migrants who repatriate gives Italy an ''unfair fiscal gain that worth around 30 percent of GDP. As most sending countries are poorer than Italy, they and the migrants in question have to care this burden. A possible solution that may help in protecting social security rights of immigrants with no bilateral agreement on social security may be the mandatory enrolment of third country nationals to pension funds (they will not pay Pay-as-you-go contributions to the Italian system). It will be a sort of pro-rata that they will obtain for sure. Another refinement could be the provision of a partial opting-out for young migrants. They can pay a contribution quota to the Italian Pay-as-you-go system and a contribution quota to an individual pension fund. Conversely, old migrants close to retirement may pay contribution to their personal account only.

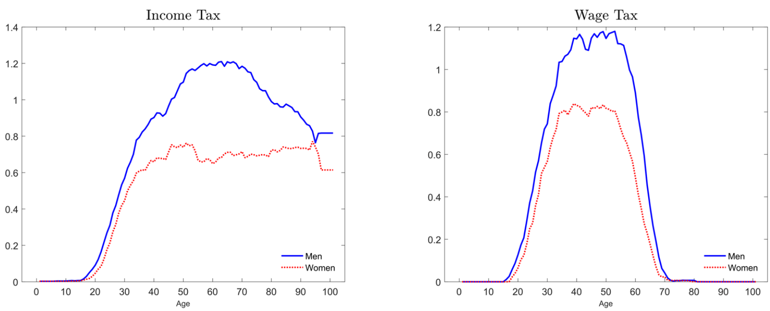
Appendix A

A.1 Micro profiles

Figure 3 reports Italian micro profiles used in the *Generational Accounting* analysis.

Figure 4 illustrates data relative to wage and employment for each foreign community of migrants, namely the Albanian, the Chinese, the Moroccan, the non-EU citizens, the Romanian, and the Ukrainian. The reported data, which are express in relation to Italian data, are rescaled with Italian micro profiles in order to derive micro profiles for migrants.

Figure 3: Italian micro profiles



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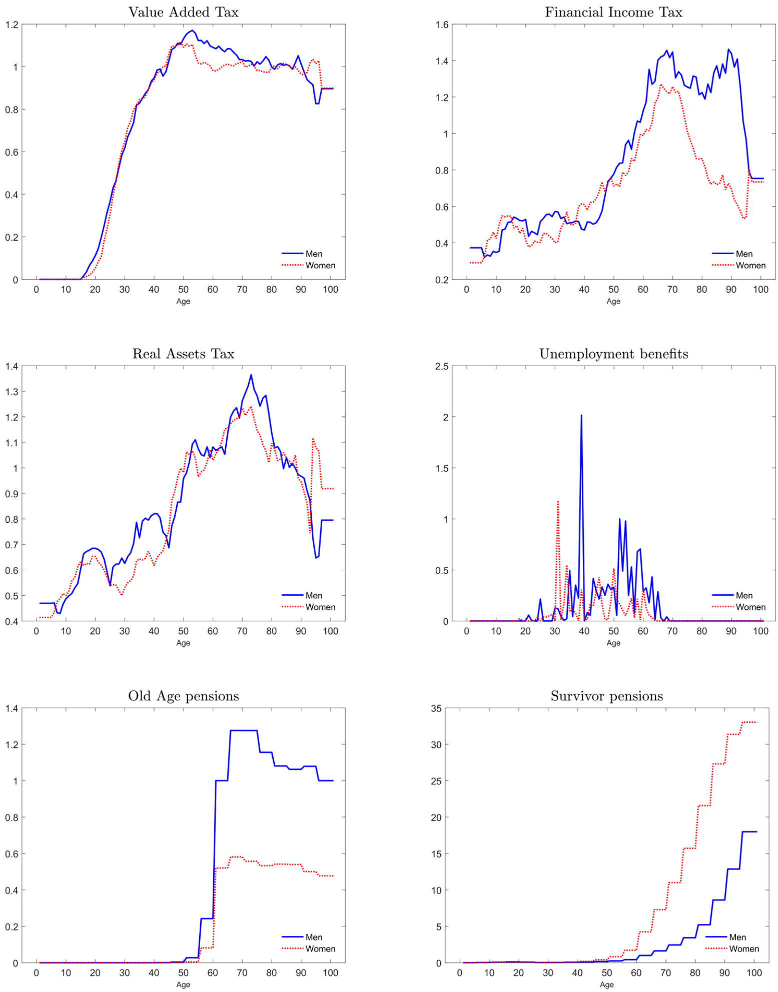

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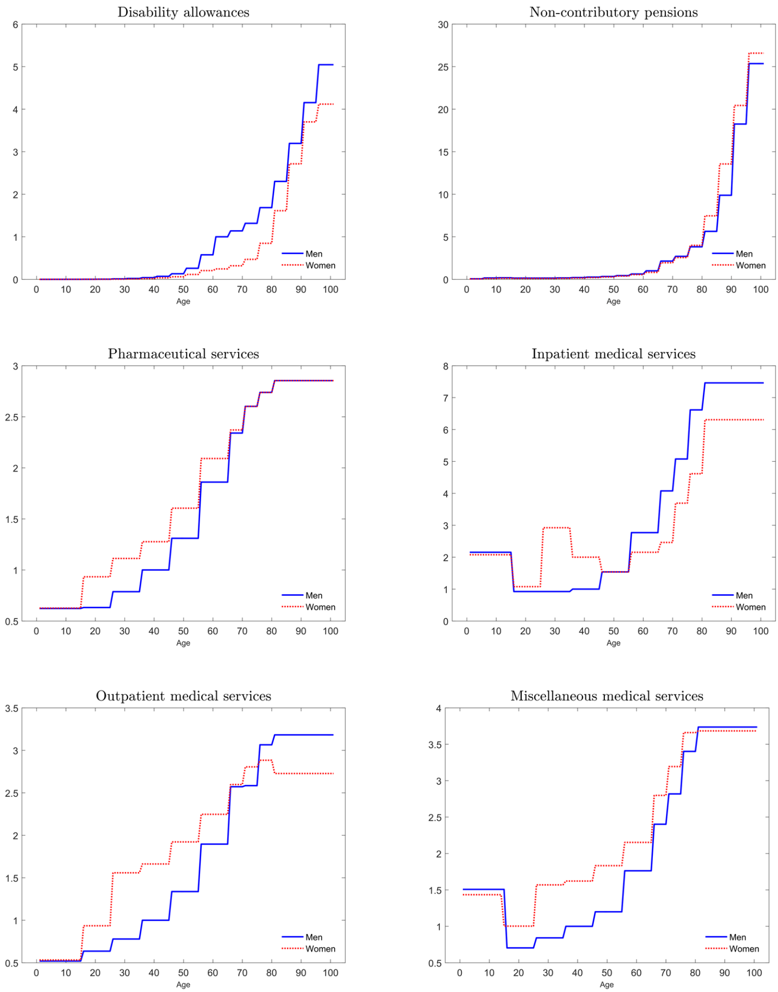

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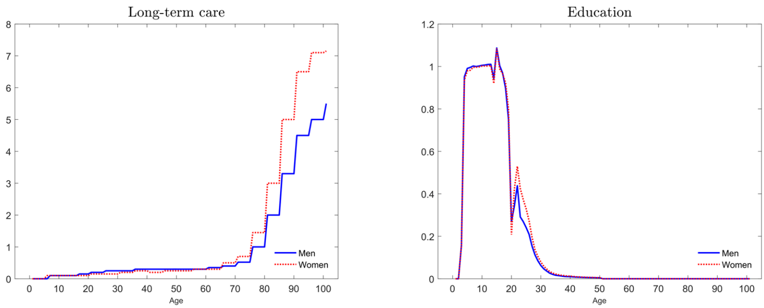
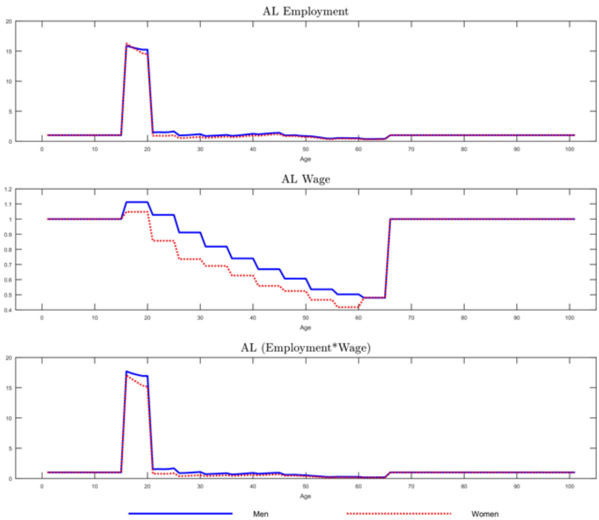


Figure 4: Employment and wage of different communities of migrants



(a) Albania

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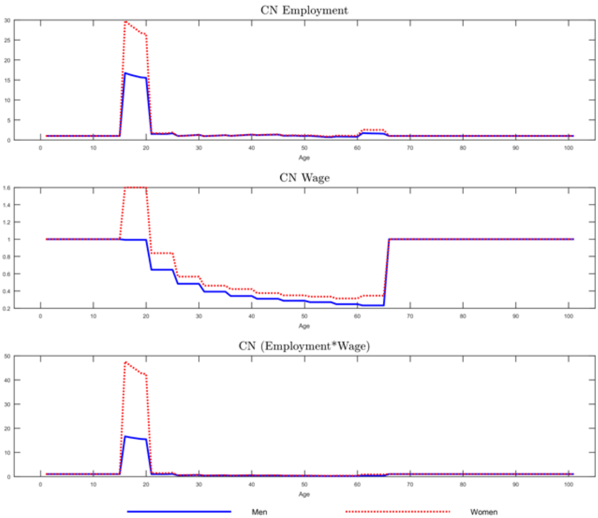
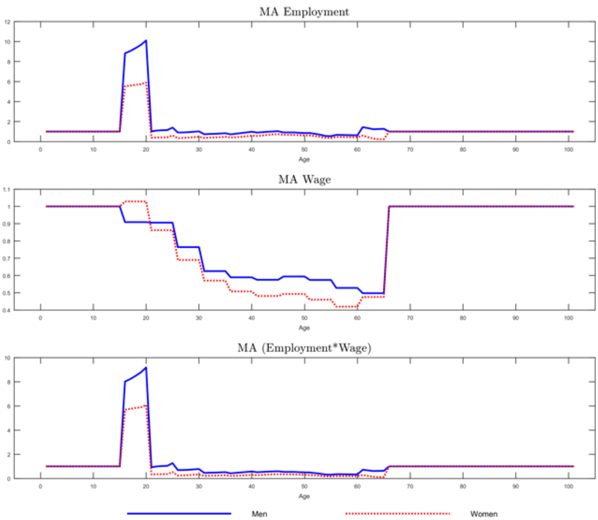


(b) China



(c) Morocco

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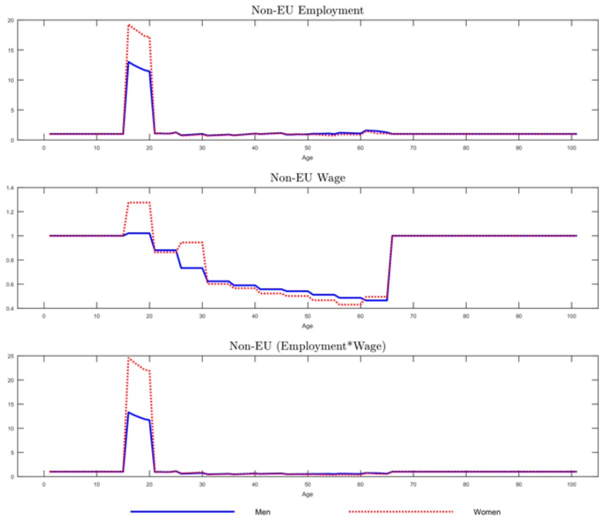
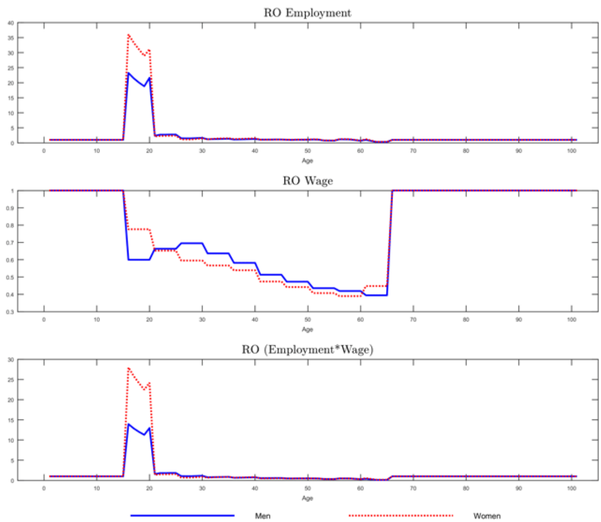


(d) Non-EU



(e) Romania

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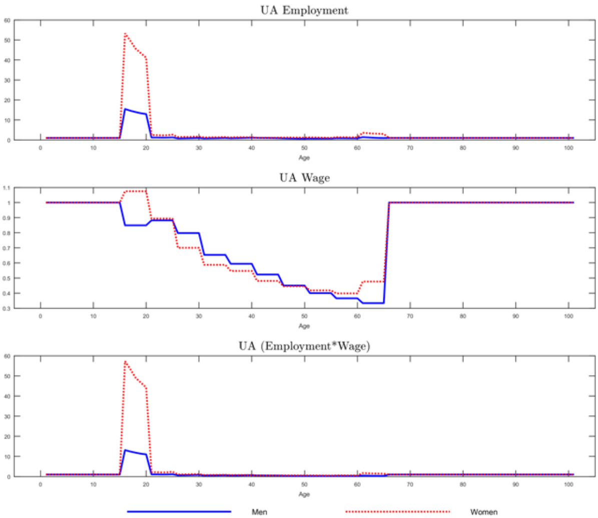
Figure 4 – *Continued from previous page*



(f) Ukraine

Chapter 3

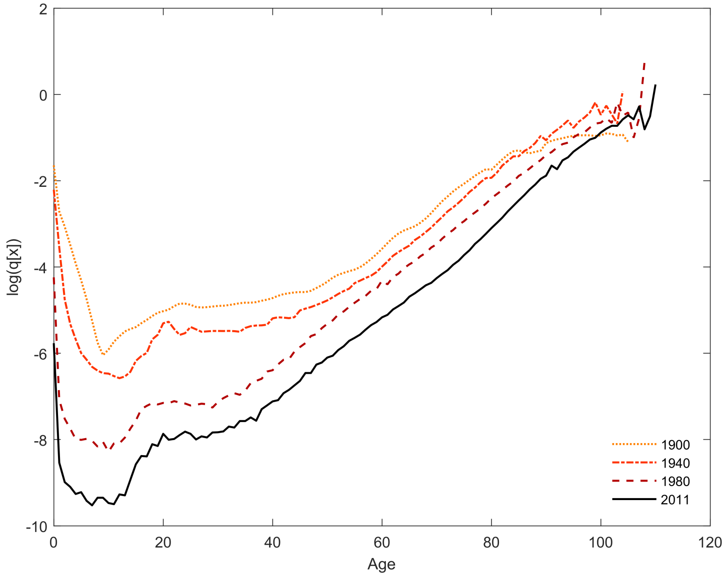
The potential costs of Longevity Risk on Public Pensions. Evidence from Italian data

3.1 Introduction

In all industrialized countries, mortality rates experienced a sharp decline during the last century (see for instance OECD (2014a)). The determinants of such decline were a decrease in infant mortality and in mortality at older ages, caused by progresses in medicine and improvements in food and living habits. This increasing longevity trend has not slowed down recently, as confirmed by recent analyses (OECD (2016)). In OECD countries, life expectancy at 65 increased from 15.8 years in 2001 to 17.6 years in 2011 for males and from 19.4 to 21 years for females. The Italian experience is in line with these observations. As Figure 5 depicts, mortality rates in Italy fell dramatically over the period 1900-2011. In the period 2001-2011, for instance, life expectancy at age 65 increased by 1.5 years for males and 1.2 years for females (OECD (2016)).

Most importantly, not only life expectancy has increased constantly

Figure 5: Italian observed log mortality rates 1900-2011



Source: HMD (2015) database

in the last century, but this rise has exceeded all expectations (IMF (2012)). No matter the approach chosen to provide forecasts, future mortality rates seem to have been constantly underestimated by models.[1] Such discrepancy between actual and expected life spans, and its related uncertainty, is called longevity risk. Indeed, while there are many positive aspects are related to the increase in the life expectancy of individuals, there are some, at present highly debated, economic issues due to

[1]Different actuarial methods can be used to obtain mortality projections (see Booth and Tickle (2008), and Pitacco et al. (2009)). Extrapolative methods, which can be either deterministic or stochastic depending on weather forecasts are built by extending past trends or by using probability distributions, make use of historical data to forecast future mortality rates, i.e. they assume that future trends will essentially be a continuation of the past. Conversely, econometric models investigate drivers of mortality (e.g. economic, environmental and epidemiological factors) in order to produce forecasts. Moreover, forecasting mortality can be based on subjective opinions of experts.

longevity risk. This risk, that is a trend risk, is relevant for individuals, companies and governments. One consequence it bears is the unexpected increase in public expenditures for the old-age, such as pensions and health expenditure. On the one hand, individuals run the risk of outliving their resources and being forced to reduce their standard of living at old ages. On the other hand, public pension schemes, as well as insurance companies and private pension funds, run the risk of paying out more than they expected to, because of unexpectedly longer life times of their insureds.

In this paper, we focus on the relevance of longevity risk for public pension providers. Indeed, they are affected by longevity risk, because they need to rely on forecasts about the future evolution of mortality rates, on which their calculations of pension benefits should be based, as highlighted by Bisetti and Favero (2014). While they are able to cope easily with the idiosyncratic, random variation risk by pooling different individuals and relying on the law of large numbers, they seem to have no strategy to defend themselves against this systematic risk. The Ageing Report of the EC (2015a) stresses how unexpected longevity gains may cause severe financial troubles, in countries where the demographic trends projected over the long term reveal a remarkable ageing process. Actually, in those countries where the old-age dependency ratio[2] is projected to increase in the future, the underestimation of mortality improvements has already exacerbated the financial burden for governments providing public pensions (generally financed via a pay-as-you-go system), forcing them to pay for retirement benefits even more than expected.[3]

In this chapter, we evaluate how uncertainty regarding future mortality and life expectancy outcomes may affect a public pension budget. We do this through an empirical analysis, based on a dataset of Italian

[2] The old age dependency ratio is the ratio of older dependents (people older than 64) to the working-age population (those aged 15-64).

[3] According to the EC (2015a), EU countries are facing a significant aging problem. As reported by Eurostat (2013) life expectancy at birth in 2060 will reach 84.8 years for males and 89.1 years for females. Such aging process leads to a substantial increase in the old-age dependency ratio, which is projected to raise from 27.8% to 50.1% over the period 2013-2060.

pensioners. Using data from WHIP (2015) - Work Histories Italian Panel, about the first state pension amounts paid to 43,641 Italians who retired between 1985-2004, we estimate the pension expenditure borne by the Italian government to pay their retirement benefits in the period 1985-2011, making use of observed mortality rates from the HMD (2015) - Human Mortality Database. Afterwards, we compare such estimation with the forecasted pension expenditure for the same sample of individuals, obtained using forecasted mortality rates. These forecasts are obtained using two well-known stochastic mortality models: the Lee and Carter (1992) and the Cairns et al. (2006) model.[4]

In particular, we first compare the actual pension expenditure with the one produced by applying the forecasts obtained using the Lee and Carter (LC from now on) model, and the historical information avilable in the base year 1984, at the beginning of the sample period. Then, as a stress, we compare the real pension expenditure with the one obtained by updating the LC forecasts every year. Finally, we compare the actual pension expenditure with the one achieved by applying the Cairns et al. model (CBD from now on) in the base year 1984.[5]

We find a consistent underestimation of improvements seen in mortality and life expectancy, when forecasts are based on expectations. The pension expenditure computed by taking into account observed mortality rates is 2.30% higher than the expenditure borne when an average scenario predicted by the LC model in the base year 1984 is used, implying that people have lived longer than expected. However, when yearly updates of the LC forecasts are employed, the underestimation reduces to 0.61%. Finally, the application of the average scenario of the CBD model leads to an underestimation of the real pension expenditure of 1.52%, one third lower than what we obtained with the LC model. Applying the CBD model widens also the variability around the central estimates, relative to the LC model.

The chapter proceeds as follows. Section 3.2 quantifies the pension

[4] For advantages and drawbacks of these two models, see for instance OECD (2014a).

[5] We apply the Cairns et al. model in the base year 1984 to forecast mortality rates for the age range 60-90. Mortality rates for the ages 0-59 and 91-110 are those obtained with the Lee and Carter model in the base year 1984.

expenditure when observed mortality rates and forecasted mortality rates based on Lee and Carter model on the base year 1984 are used. Section 3.3 discusses robustness by applying yearly updates of the Lee and Carter estimates and the Cairns et al. model to relevant ages. Finally, Section 3.4 provides some comments and concludes.

3.2 Estimating the impact of longevity risk on the Italian pension expenditure

The objective of this paper is to measure how uncertainty regarding future mortality and life expectancy outcomes have affected the Italian State budget when paying retirement benefits. We try to estimate the actual individual retirement benefits paid by the state to a sample of retirees and compare it with their forecasts at inception, according to standard stochastic mortality models. We resort to the WHIP (2015) - Work Histories Italian Panel database to have access to individual data on a sample of retirees. The WHIP database on pensions[6] covers the time span 1985-2004 with a representative 1:180 sample, keeping track of 43,641 individuals. Tables 4 and 5 summarize the composition of the sample by year of retirement, showing that the number of retirees is between 1400 and 3000 for each year.

Unfortunately, the database does not provide us with the history of the individuals in retirement, but only with the initial amount of their pension. Hence, in Section 3.2.1 we estimate the pension expenditure borne by the State for the whole sample of individuals, using the realized mortality rates of the total Italian population to reproduce the pattern of deaths in our sample, which we consider as representative of the whole retirees' population. In Section 3.2.2 we estimate the pension expenditure, according to actuarial projections regarding the development of mortality rates, based on the data available at the beginning of our observation period, 1984.

[6]The types of retirement benefit paid are: *Dipendenti, Autonomi, Agricoli, Altre gestioni, Pensioni sociali, Trattamenti diretti, Rendite Inail, Superstiti.*

Table 4: Number of pensioners by year of retirement

Year of Retirement	Number of pensioners
1985	1401
1986	1592
1987	1822
1988	1958
1989	2002
1990	2049
1991	2262
1992	2884
1993	1878
1994	2754
1995	2295
1996	2965
1997	2591
1998	2288
1999	2570
2000	2154
2001	2200
2002	1927
2003	1984
2004	2065
Total	43,641

Source: WHIP (2015) database

Table 5: Number of pensioners by year of retirement

	Year of retirement			
	1985-1989	1990-1994	1995-1999	2000-2004
Number of pensioners	8775	11,827	12,709	10,330

Source: WHIP (2015) database

3.2.1 Estimating the actual pension expenditure

To provide an estimate of the actual pension expenditure for the retirees in our sample, we use the following procedure. For each individual in our sample, retired between 1985 and 2004, we project the initial pension amount until 2011, adjusting such an amount for the annual average inflation rate.[7] We then aggregate the individual amounts by year of birth of the retirees. As a result, we obtain the pension expenditure borne up to 2011 for 83 cohorts of individuals born between 1902-1984. Then, we consider the pattern of deaths in our sample, by applying to each cohort, the observed survival rates provided by HMD (2015) for the Italian

[7] See www.inflation.eu.

population. We multiply the cohort inflated pension expenditures by these survival rates and obtain an estimate of the total pension expenditure for our sample up to 2011, aggregating across cohorts and years (1985-2011). Our estimate for the 43,641 sample individuals amounts to 6,354,610,346 Euro.

Table 6 disentangles the pension expenditure by year of retirement. The first years, even if more distant from the end of our observation period, have a relatively low expenditure due to the smaller number of individuals in the sample.

Table 6: Pension expenditure (in Euro) for sample retirees in the period 1985-2011 (by year of retirement 1985-2004)

Year of Retirement	Expenditure
1985	200,969,315
1986	234,512,955
1987	267,027,489
1988	294,324,699
1989	303,963,558
1990	328,435,410
1991	351,769,440
1992	522,936,015
1993	267,478,388
1994	508,229,840
1995	378,347,963
1996	466,544,569
1997	418,508,243
1998	313,819,978
1999	331,606,150
2000	252,950,271
2001	264,320,748
2002	227,568,902
2003	211,640,947
2004	209,655,464
Total	6,354,610,346

Source: Own calculation

3.2.2 Forecasted pension expenditure: mortality rates from the Lee and Carter (1992) model

We now want to compare our estimates of the actual pension benefits paid to our sample of retirees with some forecasts of the expenditure, based on actuarial models. These forecasts are relevant, since the pension amount granted to an individual as a public pension is not subject to revision, if the survivorship unexpectedly changes. Nonetheless, actuarial fairness would encourage to fix such pension amount appropriately, based on as-accurate-as-possible estimates of the residual lifetimes. Discrepancies between expected and realized mortality rates are exactly what we refer to as longevity risk, and determine the unexpected excess payment to pensioners in the case of improved longevity relative to forecasts. First, we produce forecasts based on 1984 data, using the most well-known stochastic mortality model, the Lee and Carter one.[8]

The Lee and Carter model (LC from now on), describes the age-period surface of log mortality-rates $log(m_{xt})$ as

$$log(m_{xt}) = a_x + b_x k_t + \epsilon_{x,t}. \tag{3.1}$$

The vector a_x can be interpreted as an average age profile, the vector k_t tracks mortality changes over time, the vector b_x determines how much each age group changes when k_t changes, and the error term $\epsilon_{x,t}$, reflects particular age-specific historical influences not captured in the model. It is well-known that the above model is over-parametrized and, thus, constraints need to be imposed in order to identify it. Following the standard identification procedure, we impose b_x's to sum to one and k_t's to sum to zero. As a consequence of this particular choice, a_x's are the average log rates.

By using Italian mortality data from 1935 to 1984,[9] we estimate a_x averaging log-rates over time and b_x and k_t via singular value decomposition of the residuals. This method approximates a matrix as the product of two vectors. However, as argued in Lee and Carter (1992), the fitted death rates derived in this way generally will not lead to the actual numbers of deaths when applied to given population age distribution. For this reason, by making use of Eq. 3.1 above, we apply a two-step estimation procedure for k_t, taking a_x and b_x estimates from the first step as given. We thereby find an estimate of k_t such that, for each

[8]Obviously, in 1984, the Lee and Carter model had not yet been formulated, but our choice of using the Lee and Carter model is justified by the fact that the Italian Statistical Institute, ISTAT, now makes use of such a model to forecast age-specific mortality rates.

[9]Even if HMD (2015) database provides Italian mortality data since 1900, we chose to discard the period 1900-1935 because it is too far away in time.

year, given the actual population age distribution, the implied number of deaths will equal the actual number of deaths. On top of estimating the model, we are interested in using it to forecast mortality. Forecasting requires the selection of the dynamics of the evolution in time of the adjusted k_t. It is usually modeled using ARIMA time series methods, and, most commonly, as a random walk with drift, i.e. an ARIMA(0,1,0). We perform the required tests and select this hypothesis to forecast our mortality rates as well, estimating appropriately the parameters of the process. Table 7 collects the whole set of estimated parameters.

We then simulate 10,000 paths of k_t for the years 1985-2011 to obtain a simulated distribution of forecasted mortality rates. Simulated k_ts are combined with the vectors a_x and b_x, estimated according to the procedure described above to produce forecasts of age-specific mortality rates for the years 1985-2011.

The fan-chart in Figure 6 illustrates the historical log mortality rates for the period 1935-1984 (HMD (2015) database), and our forecasted distribution of log mortality rates for the period 1985-2011 for a 65 years-old individual. As we can easily see from the figure, the trend of log mortality rates for future years 1985-2011 is decreasing, implying that the observed increases in life expectancy are captured by the model. However, the figure also shows the remarkable uncertainty in the evolution of mortality rates in time, especially when the objective is to forecast their value in a relatively distant future.

Having obtained a model-based forecast of the mortality rates of the Italian population using 1984 data, we can now produce the estimated forecasts of the pension expenditure of the retirees in our sample. These forecasts represents the future scenarios of expenditures that, given the pension payments granted to pensioners, the state may have expected to face according to 1984 actuarial mortality projections.[10]

As a base-line scenario, we consider the age-specific averages of our 10,000 simulations. We use these average log-mortality rates to calculate the age-specific survival rates by cohort of birth and we apply them to the

[10] Actuarial projections used for state pension calculations have usually been updated in Italy with a frequency of 8 to 10 years, thus justifying to some extent our exercise.

Table 7: Estimated parameters of the LC model

Parameters a_x and b_x

Age	a_x	b_x	Age	a_x	b_x	Age	a_x	b_x
0	-3,103	0,016	37	-6,070	0,013	74	-2,814	0,004
1	-5,339	0,031	38	-6,003	0,012	75	-2,707	0,004
2	-6,159	0,026	39	-5,942	0,011	76	-2,598	0,004
3	-6,583	0,022	40	-5,853	0,011	77	-2,496	0,004
4	-6,841	0,020	41	-5,791	0,010	78	-2,386	0,004
5	-7,033	0,019	42	-5,715	0,010	79	-2,279	0,005
6	-7,138	0,018	43	-5,641	0,009	80	-2,180	0,004
7	-7,257	0,018	44	-5,562	0,009	81	-2,086	0,004
8	-7,349	0,017	45	-5,470	0,008	82	-1,980	0,004
9	-7,406	0,017	46	-5,389	0,008	83	-1,879	0,004
10	-7,443	0,018	47	-5,310	0,007	84	-1,779	0,004
11	-7,433	0,017	48	-5,221	0,006	85	-1,693	0,004
12	-7,394	0,017	49	-5,141	0,006	86	-1,600	0,004
13	-7,324	0,016	50	-5,044	0,006	87	-1,513	0,004
14	-7,172	0,015	51	-4,968	0,005	88	-1,419	0,004
15	-7,016	0,015	52	-4,886	0,005	89	-1,322	0,004
16	-6,868	0,014	53	-4,806	0,005	90	-1,256	0,003
17	-6,767	0,014	54	-4,716	0,004	91	-1,183	0,003
18	-6,606	0,017	55	-4,637	0,004	92	-1,100	0,003
19	-6,535	0,018	56	-4,548	0,004	93	-1,024	0,003
20	-6,508	0,020	57	-4,465	0,004	94	-0,953	0,003
21	-6,478	0,020	58	-4,379	0,004	95	-0,898	0,003
22	-6,466	0,020	59	-4,290	0,004	96	-0,825	0,003
23	-6,447	0,020	60	-4,194	0,004	97	-0,774	0,002
24	-6,447	0,019	61	-4,114	0,003	98	-0,705	0,003
25	-6,445	0,019	62	-4,013	0,004	99	-0,572	0,002
26	-6,451	0,019	63	-3,925	0,004	100	-0,598	0,002
27	-6,453	0,018	64	-3,832	0,004	101	-0,557	0,002
28	-6,445	0,017	65	-3,736	0,004	102	-0,539	0,003
29	-6,428	0,017	66	-3,643	0,004	103	-0,429	0,004
30	-6,387	0,017	67	-3,545	0,004	104	-0,466	0,004
31	-6,370	0,016	68	-3,443	0,004	105	-0,223	0,009
32	-6,331	0,015	69	-3,341	0,004	106	-0,207	0,006
33	-6,287	0,015	70	-3,233	0,004	107	0,022	0,006
34	-6,241	0,014	71	-3,138	0,004	108	0,096	0,000
35	-6,190	0,014	72	-3,026	0,004	109	0,185	-0,003
36	-6,124	0,013	73	-2,921	0,004	110	0,052	-0,001

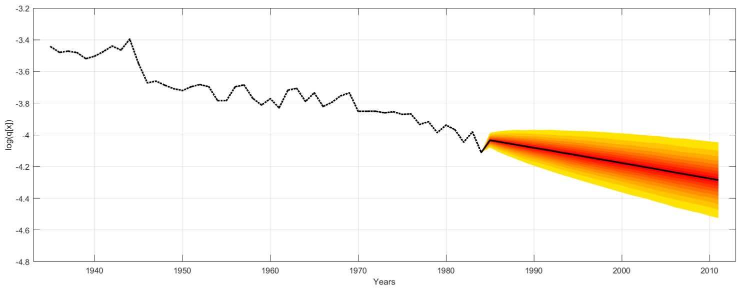
Parameter k_t ARIMA (0,1,0) Model

Drift	Variance
-2,54492	54,8979
(1.03341)	(13.0598)

Source: Own calculation

pension expenditure by cohort, as explained in the previous subsection. As a result, we obtain the total pension expenditure by year of birth, discounted by the survival rates so obtained, from the retirement date up to 2011.

Figure 6: LC forecasted log mortality rates at age 65: years 1985-2011



Source: Own calculation

As before, aggregating by cohorts and years, we obtain the total expected payment for the whole sample of pensioners up to 2011, which is 6,208,473,761 Euro. Comparing this figure with our estimate of the realized pension expenditure, we find that this estimate is 2.30% lower. This provides us with a measure of the effects of longevity risk on our sample: the best estimate of the expenditure, that is the amount that, given the profile of pension payments, the state expected to pay to retirees, was indeed about 150 million euros lower than the actual payment. This underestimation is the result of an unpredicted rise in the average life expectancy of pensioners, that even a rather sophisticated actuarial model such as the LC one could not capture.

Now, instead of taking the average of the 10,000 possible forecasted age-specific log mortality rates for the period 1985-2011, we take different percentiles of such log mortality rates. Taking a low percentile of the simulated distribution implies assuming a worst-case scenario for the annuity provider, considering that the retirees will experience mortality rates lower than the expectations. Table 8 presents the forecasted expenditures, total and by year of retirement, obtained selecting different percentiles of our simulated age-specific mortality rates, and compares them with the actual estimate presented in Section 3.2.1. Underestimation of the expenditure is obtained even when selecting a very low percentile.

At the 5^{th} percentile, representing a 1 over 20 years possible scenario, the forecasted overall expenditure is 1.04% lower than our estimate of the actual one. It is interesting to notice that this underestimation is obtained due to a too low variability of the mortality rates in the long run. Indeed, the estimates that are more distant, such as the expenditure of the 2004 retirees, are too low with respect to their actual counterparts. This is due to an excessively small variability in the forecasted mortality rates produced by the model.

Table 8: Forecasted pension expenditure (in Euro) with the LC model: base year 1984

Year	LC Average	LC 5^{th} %ile	LC 10^{th} %ile	LC 30^{th} %ile	Realized
1985	199,003,999	203,498,775	202,569,630	200,508,528	200,969,315
1986	231,506,287	236,613,552	235,557,111	233,218,222	234,512,955
1987	262,951,867	268,333,305	267,220,433	264,759,766	267,027,489
1988	289,022,079	294,798,664	293,603,204	290,965,105	294,324,699
1989	297,727,644	303,344,935	302,181,972	299,619,623	303,963,558
1990	320,982,986	326,687,311	325,506,133	322,906,110	328,435,410
1991	343,192,847	348,868,478	347,693,372	345,108,364	351,769,440
1992	510,067,308	517,470,754	515,939,554	512,569,006	522,936,015
1993	260,322,025	264,217,705	263,411,596	261,637,648	267,478,388
1994	494,472,861	500,804,655	499,497,137	496,611,973	508,229,840
1995	368,551,735	372,732,006	371,869,990	369,962,938	378,347,963
1996	454,250,861	458,986,580	458,010,043	455,847,276	466,544,569
1997	407,383,629	411,324,129	410,509,996	408,709,932	418,508,243
1998	305,938,131	308,537,065	307,999,116	306,811,993	313,819,978
1999	323,165,190	325,710,842	325,181,396	324,019,458	331,606,150
2000	246,673,676	248,419,700	248,054,577	247,258,290	252,950,271
2001	257,989,299	259,620,595	259,277,519	258,534,137	264,320,748
2002	222,486,746	223,697,787	223,442,054	222,890,445	227,568,902
2003	207,130,147	208,124,842	207,913,817	207,461,007	211,640,947
2004	205,654,444	206,466,042	206,293,604	205,924,037	209,655,464
Total	6,208,473,761	6,288,257,723	6,271,732,255	6,235,323,860	6,354,610,346
Error	2.30%	1.04%	1.30%	1.88%	-

Source: Own calculation

We now turn to analyze possible improvements of the forecasts, to see whether and how they may limit the longevity risk issue we highlighted.

3.3 Robustness

In this Section, we consider two robustness checks to our results, when improving the forecasts in two directions. First, in Subsection 3.3.1 instead of using a fixed 1984-based forecast of mortality rates for the whole period 1985-2011, we update the estimate of the mortality rate LC model. Second, in Subsection 3.3.2 we try to forecast the mortality rates of the ages 60-90 with the Cairns et al. model, that is more appropriate to describe the features of those specific ages.

3.3.1 Forecasted pension expenditure: yearly updates of LC estimates of mortality rates

As a first robustness check for our results, we update our estimates of the LC model every year starting from 1985 to 2004 (retirement window of sample retirees) and use these estimates to produce updated forecasts each year. More specifically, we use the 1984 forecast to compute the State pension expenditure borne in the period 1985-2011 for those retired in 1985. Then, we use the 1985 forecast to calculate the State expenditure of retirement benefits paid in the period 1986-2011 to those retired in 1986, and so on. This process is consistent with the idea that, in principle, a public pension provider could fix the initial pension amount for his pensioners using an actuarial fairness principle and basing the estimates of their expected residual lifetimes on information available up to the moment of retirement. Indeed, these forecasts should obviously be more precise than the ones obtained in the previous section, when the mortality model was estimated only once and for all at the beginning of the observation period.

Table 9 collects the results of this exercise. As expected, the yearly update of the estimates improves the results. However, the average total forecasted pension expenditure for the period 1985-2011 obtained by applying a yearly estimation of mortality rates amounts to 6,315,525,752 Euro, i.e. the 0.61% lower than the actual pension expenditure.

This figure is indeed closer to our estimate of the realized expenditure, but still evidences that even the yearly update of the forecasts does not immunize entirely against the risk of unexpected longevity rises. Indeed, it would be necessary to consider a "prudent" percentile of the distribution of mortality rates ranging between 10 and 30 to match the realized expenditure. This is important to keep in mind, as recent pension reforms attached the revision of pension amounts and retirement age to longevity improvements. This obviously reduces the risk of running into higher-than-predicted costs, but does not entirely avoid it.

Table 9: Forecasted pension expenditure (in Euro) with the LC model: yearly updates of estimates

Year	LC_{yu} Average	LC_{yu} 5^{th} %ile	LC_{yu} 10^{th} %ile	LC_{yu} 30^{th} %ile	Realized
1985	198,951,584	203,524,721	202,581,905	200,441,475	200,969,315
1986	231,413,279	236,573,692	235,484,186	233,153,730	234,512,955
1987	264,002,148	269,294,079	268,209,364	265,711,703	267,027,489
1988	292,029,682	297,541,217	296,361,615	293,827,020	294,324,699
1989	301,151,466	306,395,774	305,300,843	302,865,779	303,963,558
1990	326,257,527	331,475,194	330,379,981	327,967,859	328,435,410
1991	348,962,503	354,010,003	352,941,029	350,675,867	351,769,440
1992	518,593,723	525,057,801	523,575,155	520,790,381	522,936,015
1993	266,228,471	269,435,821	268,774,313	267,288,609	267,478,388
1994	505,052,213	510,120,673	508,989,100	506,743,869	508,229,840
1995	376,966,482	379,350,347	378,658,352	377,135,156	378,347,963
1996	463,818,094	467,306,918	466,510,026	464,910,444	466,544,569
1997	416,521,617	419,479,038	418,807,802	417,529,296	418,508,243
1998	312,490,960	314,382,397	313,973,988	313,133,220	313,819,978
1999	329,909,721	331,811,893	331,410,513	330,530,770	331,606,150
2000	251,997,377	253,312,423	253,020,877	252,442,193	252,950,271
2001	263,818,196	264,971,389	264,734,704	264,216,324	264,320,748
2002	227,382,182	228,208,336	228,043,633	227,656,621	227,568,902
2003	211,556,869	212,239,414	212,099,413	211,776,576	211,640,947
2004	209,321,648	209,886,047	209,771,608	209,504,059	209,655,464
Total	6,315,525,752	6,384,377,187	6,369,628,416	6,338,278,105	6,354,610,346
Error	0.61%	−0.46%	−0.23%	0.25%	-

Source: Own calculation

3.3.2 Forecasted pension expenditure: mortality rates from the Cairns et al. (2006) model

In this section, we improve our estimates by describing the evolution of mortality at ages 60-90 using the Cairns et al. model (CBD from now on). As already mentioned, differently from the LC model, the CBD model was developed with the aim of providing more accurate mortality projections for older ages, who constitute indeed the bulk of the pension expenditure of each pension system or pension fund. Indeed, according to Biffi and Clemente (2014), the CBD model is the best approach to model mortality at high ages for Italy.

According to the CBD model, the probability of death, $q_{x,t}$ is described by the following expression:

$$logit(q_{x,t}) = k_t^{[1]} + k_t^{[2]}(x - \bar{x}) + e_{x,t}. \tag{3.2}$$

The logit of the age-specific probability of death is modeled as a linear function of age x, where $\bar{x}$ represents the mean age in the sample range. The intercept $k_t^{[1]}$ and the slope $k_t^{[2]}$ are stochastic processes. We select them to be correlated random walks, as in the original formulation of the model. The former affects every age in the same way, while the impact of $k_t^{[2]}$ varies according to age, being higher for ages distant from the average age in the sample, i.e. 75. The error term $e_{x,t}$ reflects the historical (age-specific) patterns not captured by the model.
We apply this model to forecast the mortality rates of the ages 60-90 in our sample, while we maintain our original LC estimates and forecasts for the other ages. We thus estimate the model using the Italian mortality data from 1935 to 1984, estimating $k_t^{[1]}$ and $k_t^{[2]}$, whose values are collected in Table 10.

The trend of $k_t^{[1]}$ is reducing overtime, implying that the mortality rates have been decreasing at all ages. Conversely, the values of $k_t^{[2]}$ are increasing overtime, meaning that the mortality improvements have been greater at medium ages (around 60) and at higher ages rather than at central ones (around 75) in recent years.

Given the estimated parameters, we simulate $k_t^{[1]}$ and $k_t^{[2]}$ and combine them according to Eq. 3.2 to produce forecasts of mortality rates in the age range 60-90 for the period 1985-2011. The fan-chart in Figure 7 illustrates the historical log mortality rates for the period 1935-1984 (HMD database), and the 10,000 forecasted log mortality rates for the period 1985-2011 of a 65 years-old individual. One can easily see, by comparing Figure 7 and Figure 6, that the CBD model produces forecasts that are more variable than those obtained with the LC model.

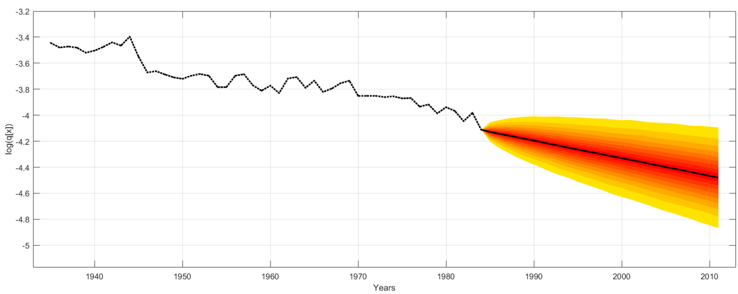
Following the same procedure described in the previous sections, we compute the pension expenditures in an average scenario and different percentile-forecasts of mortality rates. Table 11 collects the total forecasted expenditure and the forecasts for each cohort of retirees, under different mortality scenarios. In the average scenario, the forecasted ex-

Table 10: Estimated parameters $k_t^{[1]}$ and $k_t^{[2]}$ of the CBD model

Year	$k_t^{[1]}$	$k_t^{[2]}$	Year	$k_t^{[1]}$	$k_t^{[2]}$
1936	-2,343531916	0,109697942	1961	-2,712524269	0,1125323
1937	-2,337586844	0,108065883	1962	-2,628668625	0,114938986
1938	-2,328644817	0,113056448	1963	-2,631985253	0,113833747
1939	-2,342518962	0,115210143	1964	-2,717645656	0,10988918
1940	-2,283113054	0,117805985	1965	-2,660982155	0,112455753
1941	-2,279204176	0,119428198	1966	-2,735162324	0,110999325
1942	-2,248999222	0,119224772	1967	-2,719640192	0,111166669
1943	-2,265095499	0,116478356	1968	-2,681111379	0,113701263
1944	-2,242983598	0,11920428	1969	-2,691542995	0,1036612
1945	-2,377431071	0,116593622	1970	-2,750169083	0,108895078
1946	-2,500578739	0,115383117	1971	-2,767043494	0,110139067
1947	-2,505131407	0,112378237	1972	-2,789009666	0,108621377
1948	-2,548121417	0,110889088	1973	-2,748810592	0,113527407
1949	-2,537011453	0,112915383	1974	-2,797958662	0,111896011
1950	-2,603186258	0,111832384	1975	-2,759335907	0,111635557
1951	-2,508076906	0,115300696	1976	-2,784950585	0,112267495
1952	-2,518831522	0,116950192	1977	-2,813412999	0,11335119
1953	-2,514952372	0,11526673	1978	-2,843992739	0,111612077
1954	-2,635455672	0,113325938	1979	-2,871439288	0,111991897
1955	-2,642625361	0,112552946	1980	-2,857448758	0,112071886
1956	-2,496106561	0,118907274	1981	-2,882303731	0,112304065
1957	-2,580358018	0,111635482	1982	-2,928126908	0,110904174
1958	-2,666976861	0,111250593	1983	-2,894531256	0,114825542
1959	-2,708532356	0,110807465	1984	-2,97734246	0,112089278
1960	-2,652875548	0,114798641			

Source: Own calculation

Figure 7: CBD forecasted log mortality rates at age 65: years 1985-2011



Source: Own calculation

penditure for the whole sample of individuals is 6,257,819,903 Euro, 1.52% lower than the actual pension expenditure. The larger variability in the mortality dynamics captured by the CBD model relative to the

LC one leads to a better result on the tail scenarios: when the 5^{th} percentile of each age-specific mortality rate is considered, the forecasted expenditure exceeds the realized one by 0.25%.

Table 11: Forecasted pension expenditure (in Euro) with the CBD model: base year 1984

Year	CBD Average	CBD 5^{th} %ile	CBD 10^{th} %ile	CBD 30^{th} %ile	Realized
1985	201,105,097	207,877,143	206,499,849	203,400,992	200,969,315
1986	233,962,706	241,630,518	240,071,462	236,564,002	234,512,955
1987	265,636,855	273,590,700	271,975,438	268,340,381	267,027,489
1988	292,009,424	300,514,859	298,789,413	294,902,526	294,324,699
1989	300,732,432	308,928,064	307,265,183	303,522,516	303,963,558
1990	324,147,870	332,387,642	330,715,271	326,955,491	328,435,410
1991	346,484,665	354,595,258	352,949,317	349,250,787	351,769,440
1992	514,543,141	524,874,894	522,777,993	518,070,314	522,936,015
1993	262,709,730	268,272,753	267,144,008	264,605,817	267,478,388
1994	498,748,113	507,504,533	505,725,706	501,734,426	508,229,840
1995	371,387,905	377,092,234	375,931,738	373,331,190	378,347,963
1996	457,652,533	464,056,633	462,752,553	459,831,409	466,544,569
1997	410,312,806	415,662,556	414,570,586	412,128,305	418,508,243
1998	307,873,733	311,352,741	310,640,576	309,052,233	313,819,978
1999	325,152,307	328,577,106	327,874,216	326,308,289	331,606,150
2000	248,101,720	250,443,955	249,961,761	248,889,815	252,950,271
2001	259,367,580	261,546,291	261,096,104	260,098,145	264,320,748
2002	223,505,240	225,105,233	224,773,223	224,039,984	227,568,902
2003	208,025,096	209,342,129	209,067,703	208,463,912	211,640,947
2004	206,360,949	207,412,550	207,191,488	206,710,370	209,655,464
Totals	6,257,819,903	6,370,767,792	6,347,773,584	6,296,200,901	6,354,610,346
Error	1.52%	-0.25%	0.11%	0.92%	-

Source: Own calculation

3.4 Conclusions

In this chapter we provided evidence about the relevance of longevity risk for public pension issuers. Based on Italian data, we quantified a severe underestimation of mortality rates over a 30 year span, when forecasts are based on central estimates provided by sound actuarial stochastic mortality models. Our results show that, in our sample, not only actual pension payments have exceeded expectations, but that actual expenditure turned out to be a very low percentile of the forecasted distribution.

Our real-world based experiment clarifies the importance of considering the uncertainty in predictions. Even though longevity is perceived as a threat that builds slowly over time, it is now general consensus that it needs to be tackled, to prevent further deterioration of countries financial stability. Measures able to avoid potential large negative effects on private and public sector finances are needed. The debate on how to reduce the risks related to aging and to the underestimation of longevity improvements for public pensions has recently gained attention. IMF (2012), for instance, proposes several potential solutions, aimed at reducing the threat that longevity risk represents for financial viability. First, longevity risk, to which public pension systems are highly exposed, as we showed in our results, can be shared to some extent between the private sector, the public sector, and individuals, through pension reforms. An example of such sharing mechanism is retirement age indexation to life expectancy improvements, that has been adopted recently by some countries, such as Italy. A more complex, but effective, solution may consist in allowing governments, individuals and private pension providers to transfer the longevity risk to capital market participants and private companies (see for instance Blake and Burrows (2001) and Blake et al. (2014)). However, most importantly, a careful investigation about the uncertainty regarding mortality rates forecasts is necessary. A correct, threat-minimizing valuation of pension benefits appears to be crucial for public pension systems, just as well as for private ones, in reducing financial instability issues.

Chapter 4

Intragenerational Redistribution in a Funded pension system

4.1 Introduction

According to OECD (2014b) and OECD (2015), many OECD countries are currently facing a remarkable aging process which is driven by increases in life expectancy and declines in fertility rates. The observed increases in the median age of the population, which is projected to continue raising over the years ahead, have resulted in an increase in the old-age dependency ratio.[1] Actually, according to OECD (2015), the demographic dependency ratio, equal to 14 in 1950, reached the value of 28 in 2015, and it is projected to nearly double in 2075 reaching the value of 55.

The increase in the old-age dependency ratio should be seriously taken into account by governments who are financing public pensions via the Pay-as-you-go system (PAYG). In a PAYG system, active population fi-

[1] The demographic old-age dependency ratio is defined as the number of individuals aged 65 and over per 100 people of working age defined as those aged between 20 and 64 OECD (2015)).

nance pensions of same-period retirees based on the promise that they will receive a similar treatment by future workers. Due to this working mechanism, a decline in population growth may jeopardize the financial viability of the system itself since it reduces the likelihood that the promise can be maintained in the future.[2] Actually, many OECD countries providing PAYG-financed public pensions will experience financial troubles caused by the retirement of the Baby Boom generation, which is a cohort much larger than the one that followed in the workforce.

According to World Bank (2005), traditional strategies adopted in order to make public pension promises more affordable within a PAYG system, i.e. adjustments in the pension eligibility age, in indexation arrangements and in the benefits accrual rate, are often proved unsatisfactory.

In search for sustainability, some countries (e.g., Italy and Sweden) have introduced the Notional defined contribution (NDC) scheme. By maintaining a PAYG-finance, the NDC plan provides benefits that bear an actuarial relationship to individual lifetime contributions.[3] Nevertheless, due to demographic changes, the introduction of the NDC can only partially ease PAYG system's sustainability tension.

Therefore, in a context of falling fertility and rising longevity one should seriously consider not merely an amendment of the system but a replacement of such a system with a more sustainable and a fair one.

The most credited solution to the solvency problem originated by PAYG system in an aging economy, is the privatization of social security, i.e. the shift from the PAYG to the Fully-funded system (FF) where each individual builds up her own pension by contributing to a personal account.[4] However, the adoption of a FF system does not come with dis-

[2] According to Samuelson (1958), Aaron (1966) and Samuelson (1975), the PAYG system, defined as the social contract between generations, is desirable only when each generations maintain positive real rates of return on contributions, which happens so long as real earnings growth and population growth remain positive.

[3] See for example Disney et al. (1999), Holzmann et al. (2006) and World Bank (2005).

[4] We refer to FF defined contribution schemes where the post-retirement consumption, given life expectancy and the interest rate, is determined only by the amount of contributions paid into the fund. In an aging economy the FF defined benefit scheme, that precommit to pay a defined pension benefit no matter on the value of assets accumulated, runs

advantages and for this reason there is no common opinion in the literature on whether PAYG systems should be entirely replaced with funded systems.[5]

In particular, in addition to the financial viability of the pension system, which has become of a primary importance, any social welfare program has to guarantee an adequate standard of living in retirement (OECD (2014b)).

In this regard, the FF system is able to improve financial sustainability but, due to its working mechanism and contrarily to unfunded schemes, it does not accomplish poverty relief and redistribution,[6] which are two of the most important objectives pursued by social security systems.[7] Actually, the financial unsustainable PAYG system is able to ensure an adequate income throughout retirement by including different types of redistribution, namely the intergenerational and the intragenerational one.[8] The former redistributes resources across generations (e.g. a reduction in the contribution rate of current workers requires an increase in contributions paid by future generations or a decrease in the amount of their pension benefits). The latter, instead, allocates resources across different income levels within the same generation (e.g., by guaranteeing a higher replacement rate to low income earners). Similarly, the NDC, by maintaining a PAYG finance, provide intergenerational redistribution and very slight forms of within-cohort redistribution, namely the one from men to women, the one achieved through survivor benefits, and the one obtained through credits for periods spent out of the paid labor force.[9]

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the solvency risk (OECD (2014b)).

[5] Among others, see Sinn (2000) for a general comparison between funded and unfunded systems.

[6] Blake (2006) argues that even the FF provides slight forms of redistribution. In defined benefit plans early leavers subsidize long stayers, while in defined contribution schemes poor people subsidize rich people, and (if there is a unisex annuity rate) men subsidize women.

[7] See Barr and Diamond (2006).

[8] See Disney (1996), and Ignacio Conde-Ruiz and Profeta (2007).

[9] According to World Bank (2005), the redistribution from men to women arises from the use of unisex annuity factors even though women tend to live longer. Conversely, the redistribution via survivor benefits occurs because annuity factors do not include the

In the attempt to overcome the solvency problems induced by PAYG, and to pursue the objective of retirement-income adequacy unmet by the FF, we introduce a modified version of the pure FF system that incorporates an intragenerational redistributive component.

To analyze the effects of such modification, a general equilibrium, two-period overlapping generations model is developed. Individuals decide how much to work and how much to save for old age consumption. The returns to private savings and wages are determined by the profit maximizing behavior of a firm with standard Cobb-Douglas production function. Since a representative agent framework does not capture intragenerational redistribution, we present a model with two productivity types. Differences in the skill level (and thus in the income level) among workers translate into differences in the magnitude of capital accumulation and labor supply distortion.

According to our results, the unfunded system generates labor supply distortions and depresses physical capital accumulation. More precisely, in the numerical investigation of the theoretical model we propose, the labor distortion in the PAYG NDC arises from government's decision to revaluate workers contributions at a rate that is lower than the market interest rate.[10] Therefore, by offering a lower amount of labor, the individual depresses his income and his savings. At the equilibrium, this reduction in saving causes the aggregate capital stock to fall. Conversely, the pure funded system depresses neither labor supply nor capital accumulation since individuals recognize that contributions paid during the entire working career are a form of private savings. The most interesting result of our model is that the intragenerational redistribution carried out in the modified version of the original funded system slightly increase physical capital accumulation with respect to the pure FF case, without significantly reducing labor supply incentives. Hence, the pro-

number and age of dependents.

[10]Even though the capital stock in the balanced growth path of the Diamond (1965) model may exceed the golden-rule level, implying a dynamically inefficient economy, the realistic parameter values we chose in our numerical simulation ensure that the interest rate is always greater than the PAYG NDC returns, implying a dynamically efficient economy.

posed modified funded pension system, which incorporates a redistribution share aimed at contributing to the creation of minimum living standards pillar to low-income pensioners, seems to be a good compromise to pursue the objectives of reducing labor supply distortions and enhancing physical capital accumulation.

The chapter is organized as follows. The model is presented in Section 4.2. Section 4.3 provides the closed economy general equilibrium analysis alongside the main analytical findings. Section 4.4 completes the analysis with a numerical investigation and Section 4.5 provides conclusions.

4.2 The Model

We present a two-period overlapping generations (OLG) model à la Diamond (1965) with endogenous labor supply to study the impact of different pension schemes.[11] We consider a PAYG NDC in opposition to a Modified Fully Funded (MFF) scheme, a funded system that embodies a redistributive component. We consider a closed economy where firms produce a single homogeneous good that can be used for both consumption and investment. Moreover, capital and labor are used as inputs in a constant returns Cobb-Douglas technology. Finally, individuals differ in their skill level.

This section starts with Subsection 4.2.1 on individuals optimal saving and labour supply decisions, while in Subsection 4.2.2 the production function of the economy is presented. Lastly, in Subsection 4.2.3 we analyze the public budget constraint under the two aforementioned pension systems.

4.2.1 Households Decision

In our OLG setting people work in the first period of their life and retire in the second. As usual, the population growth rate is equal to $\frac{N_{t+1}}{N_t} = 1 + \rho_{t+1}$ with $\rho_{t+1} > -1$, where N_t denotes the population at time t. Moreover,

[11] See also Breyer and Straub (1993) and Sommacal (2006).

by assuming that generations are non-altruistic, we rule out bequests. Individuals differ in their productivity level h_i, which can be either high or low, i.e. $i = H, L$. In particular, we assume that a fixed fraction $\lambda_L \in (0,1)$ of the total population belongs to the low skilled class, while the other fixed fraction $\lambda_H = 1 - \lambda_L$ belongs to the high skilled class. The income level of an individual i in his working period t ($y_{i,t}$) is equal to $w_t h_i l_{i,t}$ where w_t is the wage rate per efficient unit of labor and $l_{i,t}$ is the labor supply provided in the same working period by an individual of type i. In our analysis, it is assumed that $0 < h_L < h_H$. Both types of individuals contribute to the public pension system when young and receive pension benefits when retired. The pension contribution rate is exogenous and equal to τ where $0 < \tau < 1$.

To keep things tractable, we take a convenient and quite standard specification for the utility function: we assume it quasilinear in labor.[12] Consequently, the individual maximization problem becomes:

$$\begin{aligned} \max_{l_{i,t}, s_{i,t}, c_{i,t}, c_{i,t+1}} \quad & U_{i,t} = ln\left(c_{i,t} - \frac{l_{i,t}^2}{2}\right) + \beta ln c_{i,t+1} \\ \text{s.t.} \quad & c_{i,t} + s_{i,t} = (1-\tau) w_t h_i l_{i,t} \\ & c_{i,t+1} = R_{t+1} s_{i,t} + p_{i,t+1} \end{aligned} \tag{4.1}$$

In the first period the young working generations allocate their after tax wage income $(1-\tau) w_t h_i l_{i,t}$ between consumption $c_{i,t}$ and savings $s_{i,t}$. The old retired generation receives their previous savings $s_{i,t}$ plus the return r_{t+1}, where $R_{t+1} = 1 + r_{t+1}$, and a retirement benefit $p_{i,t+1}$, which depends on the pension system in force. The parameter $\beta \in [0,1]$ represents the preference for future consumption for each type of individuals.

We assume that in the first period, when individuals make labor supply and saving decisions, the variables R_{t+1} and w_t are perfectly known to consumers. Conversely, as it will be clear in Subsection 4.2.3, the dependence of $p_{i,t+1}$, on the decision variable $l_{i,t}$ varies according to the existing pension system.

[12]The choice of a quasilinear utility function imposes some restrictions since it implies no income effects. The choice of a quadratic form of labor disutility is introduced to keep the problem simple.

Concluding, given the own specific productivity level h_i, each individual chooses $l_{i,t}, s_{i,t}, c_{i,t}, c_{i,t+1}$ to maximize his life-cycle utility under the constraints in Eq. 4.1. The solutions to this optimization problem are:

$$s_{i,t} = \frac{\beta(1-\tau)w_t h_i l_{i,t} - \frac{\beta l_{i,t}^2}{2} - \frac{p_{i,t+1}}{R_{t+1}}}{1+\beta} \tag{4.2}$$

$$l_{i,t} = (1-\tau)w_t h_i + \frac{\frac{\partial p_{i,t+1}}{\partial l_{i,t}}}{R_{t+1}} \tag{4.3}$$

Notice that both the decisions for savings $s_{i,t}$ and for labor supply $l_{i,t}$ depend on the pension system in force (e.g., through the terms $p_{i,t+1}$ and $\frac{\partial p_{i,t+1}}{\partial l_{i,t}}$, which will be discussed in detail in Subsection 4.2.3).

4.2.2 Production

Firms produce a single homogeneous good according to a Cobb-Douglas technology exhibiting constant returns to scale. Therefore, the production function $F(K,L)$ is:

$$F(K_t, L_t) = AK_t^{\alpha} L_t^{1-\alpha} \tag{4.4}$$

where K is the aggregate capital stock, L is the aggregate labor input and $\alpha \in (0,1)$.

Output and factor markets are competitive, which implies that firms hire physical capital and labor until gross factor prices equal marginal products:

$$w_t = A(1-\alpha)k_t^{\alpha} \tag{4.5}$$

$$R_t = (1+r_t) = A\alpha k_t^{\alpha-1} \tag{4.6}$$

where w_t is the wage rate in period t.

The labor market clearing condition yields:

$$L_t = N_t(\lambda_L h_L l_{L,t} + \lambda_H h_H l_{H,t}) \tag{4.7}$$

We denote by $k_t = \frac{K_t}{L_t}$ the capital per efficiency unit, which we can express as:

$$k_t = \frac{K_t}{L_t} = \frac{K_t}{N_t(\lambda_L h_L l_{L,t} + \lambda_H h_H l_{H,t})} \tag{4.8}$$

Finally, we define the capital per worker as:

$$\frac{K_t}{N_t} = k_t(\lambda_L h_L l_{L,t} + \lambda_H h_H l_{H,t}) \tag{4.9}$$

4.2.3 Pension Systems and Government Budget Constraint

We compare, in terms of labor market distortions and physical capital accumulation, a PAYG NDC[13] and a funded system that embodies an intragenerational component, i.e. the MFF scheme.

Pay-as-you-go Notional Defined Contribution (PAYG NDC) System
In PAYG, pensions are paid out of contributions of current workers. Therefore, since the State can directly tax the working population to finance the pensions of the retired generation, there is no need to accumulate assets in anticipation of future pension claims.

Consequently, the government budget constraint is balanced when the following equality holds:

$$\begin{aligned}\lambda_L p_{L,t+1}^{PAYG\ NDC} + \lambda_H p_{H,t+1}^{PAYG\ NDC} = \\ &\tau w_{t+1}^{PAYG\ NDC} \\ &\cdot (\lambda_L h_L l_{L,t+1}^{PAYG\ NDC} + \lambda_H h_H l_{H,t+1}^{PAYG\ NDC}) \\ &\cdot (1 + \rho_{t+1})\end{aligned} \tag{4.10}$$

Regarding the link between contributions and benefits, the key characteristics of a PAYG NDC system are the payment of a pension whose present value depends entirely on the individual's contribution history,

[13] A preliminary version of the PAYG system appeared in Wen et al. (2015).

and the application of an interest rate set by government rules. As a result, the pension payments rule for low skilled and high skilled workers becomes:

$$\frac{p_{L,t+1}^{PAYG\ NDC}}{p_{H,t+1}^{PAYG\ NDC}} = \frac{h_L l_{L,t}^{PAYG\ NDC}}{h_H l_{H,t}^{PAYG\ NDC}} \tag{4.11}$$

Accordingly, combining Eqs. 4.10 and 4.11, the pension benefits for a type i agent under this pension system are expressed as follows:

$$p_{i,t+1}^{PAYG\ NDC} = \tau w_t^{PAYG\ NDC} h_i l_{i,t}^{PAYG\ NDC} (1 + \rho_{t+1}) \Omega_{t+1}^{PAYG\ NDC} \tag{4.12}$$

where $\Omega_{t+1}^{PAYG\ NDC}$ denotes the growth factor of the economy's per capita income at time $t+1$, which is defined as:

$$\Omega_{t+1}^{PAYG\ NDC} = \frac{w_{t+1}^{PAYG\ NDC}(\lambda_L h_L l_{L,t+1}^{PAYG\ NDC} + \lambda_H h_H l_{H,t+1}^{PAYG\ NDC})}{w_t^{PAYG\ NDC}(\lambda_L h_L l_{L,t+1}^{PAYG\ NDC} + \lambda_H h_H l_{H,t+1}^{PAYG\ NDC})} \tag{4.13}$$

Fully Funded (FF) and Modified Fully Funded (MFF) Systems
Differently from PAYG NDC, in FF pensions are paid out of a fund built over a period of years from its members contributions. Contributions are invested in financial or real assets, the return on which is credited to the fund. The main feature of this funded system is that it does not allow for redistribution across generations since the position of each cohort is determined by its own past savings. Moreover, without any provision for within-cohort redistribution, at retirement each individual obtains no more than his first period savings and his past contributions, together with the return yielded by the fund. Summing up, in a FF system, the government's budget is balanced when:

$$\lambda_L p_{L,t+1}^{FF} + \lambda_H p_{H,t+1}^{FF} = R_{t+1}^{FF} \tau w_t^{FF} (\lambda_L h_L l_{L,t}^{FF} + \lambda_H h_H l_{H,t}^{FF}) \tag{4.14}$$

Moreover, the pension payments rule for the low skilled and high skilled workers is similar to Eq. 4.11, and is expressed as:

$$\frac{p^{FF}_{L,t+1}}{p^{FF}_{H,t+1}} = \frac{h_L l^{FF}_{L,t}}{h_H l^{FF}_{H,t}} \tag{4.15}$$

Accordingly, by combining Eqs. 4.14 and 4.15, the pension benefits for a type i agent under this system are expressed as follows:

$$p^{FF}_{i,t+1} = \tau w^{FF}_t h_i l^{FF}_{i,t} R^{FF}_{t+1} \tag{4.16}$$

The absence of redistribution across individuals, and thus the difficulties in pursuing poverty relief during retirement, give us room for proposing a modified version of the classical well-known FF system. The refinement of the original version that we introduce has the purpose to improve the living conditions for the low skilled retirees, who may not save enough or accumulate enough in their fund for the retirement period.

The Modified Fully Funded (MFF), works as the FF but in addition disposes a lump-sum withdrawal $b \in [0,1]$ from every individuals account before the beginning of the retirement period. The resources collected in this way are invested in a parallel fund that is used for redistribution once the same individuals become eligible for qualifying themselves as retirees.

Therefore, at the time of retirement every individual receives a pension which is made of two components:

$$p^{MFF}_{i,t+1} = (1-b)p^{FF}_{i,t+1} + bp^{R}_{i,t+1} \tag{4.17}$$

The first part of the pension obtained in the MFF system, namely $(1-b)p^{FF}_{i,t+1}$, is the usual FF pension from which the withdrawal b is taken away, while the second part, corresponding to $bp^{R}_{i,t+1}$, where R stands for redistribution, represents the additional benefit given to every individual irrespective of the skill/income level, i.e. the benefit needed to carry on intragenerational redistribution. Therefore, by definition, the redistributive component of $p_{i,t+1}^{MFF}$ obeys the rule[14]:

$$p_{L,t+1}^{R} = p_{H,t+1}^{R} \tag{4.18}$$

which simply remarks that the resources in the aforementioned parallel fund are spread equally among the whole population irrespective of the belonging income class and thus irrespective of the contributions one made when young.

Concluding, the two components of $p_{i,t+1}^{MFF}$ satisfy:

$$p_{i,t+1}^{FF} = R_{t+1}^{MFF} \tau w_t^{MFF} h_i l_{i,t}^{MFF} \tag{4.19}$$

and:

$$p_{i,t+1}^{R} = R_{t+1}^{MFF} \sum_{i=L,H} \lambda_i \tau w_t^{MFF} h_i l_{i,t}^{MFF} \tag{4.20}$$

4.3 General Equilibrium

The following definition introduces the general equilibrium given the different social security systems.

Definition 1. *Given the state of agents distribution in the economy and the productivity levels of low skilled and high skilled individuals, a general equilibrium is a sequence of individuals decisions, a sequence of factor prices, and a sequence of pension payments so that:*

1. *Individuals choose $l_{i,t}, s_{i,t}, c_{i,t}, c_{i,t+1}$ to solve the maximization problem described by Eq. 4.1, taking the factor prices as given;*

2. *Factor markets clearing condition holds: the factor prices are equal to their marginal products, see Eqs. 4.5 and 4.6;*

3. *The government budget constraint is satisfied, i.e., Eqs. 4.12 and 4.13 are satisfied in the PAYG NDC system, while Eqs. 4.17-4.20 are satisfied in the MFF system.*

[14]Notice that the other component of Eq. 4.17 satisfies Eq. 4.15. Clearly, when $b = 0$ there is no withdrawal and the system becomes a pure FF.

In particular, in an unfunded system, the supply of capital in period $t+1$ is determined by the saving decision of the young in period t. Hence, physical capital $K_{t+1}^{PAYG\ NDC}$ in period $t+1$ is the sum of the aggregate previous period private savings:

$$K_{t+1}^{PAYG\ NDC} = \sum_{i=L,H} \lambda_i N_t s_{i,t}^{PAYG\ NDC} \tag{4.21}$$

where $s_{i,t}$ is provided by Eq. 4.2.

On the other hand, in a funded system, the aggregate physical capital consists both of private and public savings of the former period. Accordingly, the aggregate capital in the MFF case is:

$$K_{t+1}^{MFF} = \sum_{i=L,H} (\lambda_i N_t s_{i,t}^{MFF} + \lambda_i N_t \tau w_t^{MFF} h_i l_{i,t}^{MFF}) \tag{4.22}$$

4.3.1 General Equilibrium in the PAYG NDC System

We find the general equilibrium for the PAYG NDC system and summarize the result in Preposition 1 (details of derivation are provided in the Appendix B). In part 1 we derive the labor supply choice $l_{i,t}^{PAYG\ NDC}$ of both high and low skilled workers, which holds at any time t. In part 2 we express, for each time stage t, the just mentioned optimal labor supply as a function of the capital per efficiency unit $k_t^{PAYG\ NDC}$. In part 3, we provide a recursive way to compute $k_{t+1}^{PAYG\ NDC}$ knowing $k_t^{PAYG\ NDC}$. Finally, in part 4, we show that, when the population growth rate is constant and equal to $1+\rho$, one obtains a unique nontrivial steady state solution (i.e., one characterized by non-zero values of individual labor supply choices, of capital per efficiency unit, and of capital per worker).

Preposition 1 [PAYG NDC]. The following holds for the general equilibrium of the PAYG NDC system with a quasilinear specification of the utility function.

1. Relationship between individual labor supply choice and capital per efficiency unit:

$$l_{i,t}^{PAYG\,NDC} = \frac{[\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1-\alpha)(1-\tau)Ah_i}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)} \cdot (k_t^{PAYG\,NDC})^{\alpha} \tag{4.23}$$

2. Relationship between labor supply choices of high and low skilled individuals:

$$l_{H,t}^{PAYG\,NDC} = \frac{h_H}{h_L} l_{L,t}^{PAYG\,NDC} \tag{4.24}$$

3. Recursive formula for the capital per efficiency unit:

$$k_{t+1}^{PAYG\,NDC} = \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho_{t+1})}\right]^{\frac{1}{1+\alpha}} \cdot (k_t^{PAYG\,NDC})^{\frac{2\alpha}{1+\alpha}} \tag{4.25}$$

4. Unique nontrivial steady state solution ($\rho_t = \rho$ for all t):

$$k^{PAYG\,NDC} = \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{1}{1-\alpha}} \tag{4.26}$$

$$l_i^{PAYG\,NDC} = (1-\tau)A(1-\alpha)\frac{\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)} \cdot \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} h_i \tag{4.27}$$

$$\frac{K_t^{PAYG\,NDC}}{N_t} = (\lambda_L h_L^2 + \lambda_H h_H^2)(1-\tau)A(1-\alpha) \cdot \frac{\beta\tau(1-\alpha) + 2\alpha(1+\beta) + \tau(1-\alpha)(2+\beta)}{2\alpha(1+\beta) + \tau(1-\alpha)(2+\beta)} \cdot \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta) + \tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{1+\alpha}{1-\alpha}} h_i \tag{4.28}$$

4.3.2 General Equilibrium in the MFF System

Preposition 2 summarizes the general equilibrium in a MFF system (details of derivation are provided in the Appendix B). For this preposition, one can make comments similar to those stated before Preposition 1.

Preposition 2 [MFF]. The following holds for the general equilibrium of the MFF system with a quasilinear specification of the utility function.

1. Relationship between individual labor supply choice and capital per efficiency unit:

$$l_{i,t}^{MFF} = (1 - b\tau(1-\lambda_i))A(1-\alpha)h_i(k_t^{MFF})^{\alpha} \tag{4.29}$$

2. Relationship between labor supply choices of high and low skilled individuals:

$$l_{H,t}^{MFF} = \frac{(1-b\tau\lambda_L)h_H}{(1-b\tau\lambda_H)h_L} l_{L,t}^{MFF} \tag{4.30}$$

3. Recursive formula for the capital per efficiency unit:

$$k_{t+1}^{MFF} = \left[\frac{A\beta[\lambda_L h_L^2 + \lambda_H h_H^2 - b^2\tau^2(\lambda_H h_L^2 + \lambda_L h_H^2)\lambda_L\lambda_H](1-\alpha)}{2(1+\beta)(1+\rho_{t+1})[\lambda_L h_L^2 + \lambda_H h_H^2] - b\tau(h_L^2 + h_H^2)\lambda_L\lambda_H}\right]^{\frac{1}{1+\alpha}} \cdot (k_t^{MFF})^{\frac{2\alpha}{1+\alpha}} \tag{4.31}$$

4. Unique nontrivial steady state solution ($\rho_t = \rho$ for all t):

$$k^{MFF} = \left[\frac{A\beta[\lambda_L h_L^2 + \lambda_H h_H^2 - b^2\tau^2(\lambda_H h_L^2 + \lambda_L h_H^2)\lambda_L\lambda_H](1-\alpha)}{2(1+\beta)(1+\rho)[\lambda_L h_L^2 + \lambda_H h_H^2] - b\tau(h_L^2 + h_H^2)\lambda_L\lambda_H} \right]^{\frac{1}{1-\alpha}} \tag{4.32}$$

$$l_i^{MFF} = (1 - b\tau(1-\lambda_i))A(1-\alpha) \left[\frac{A\beta[\lambda_L h_L^2 + \lambda_H h_H^2 - b^2\tau^2(\lambda_H h_L^2 + \lambda_L h_H^2)\lambda_L\lambda_H](1-\alpha)}{2(1+\beta)(1+\rho)[\lambda_L h_L^2 + \lambda_H h_H^2] - b\tau(h_L^2 + h_H^2)\lambda_L\lambda_H} \right]^{\frac{\alpha}{1-\alpha}} \tag{4.33}$$

$$\frac{K_t^{MFF}}{N_t} = [\lambda_L h_L^2 + \lambda_H h_H^2 - b\tau\lambda_L\lambda_H(h_L^2 + h_H^2)]A(1-\alpha) \left[\frac{A\beta[\lambda_L h_L^2 + \lambda_H h_H^2 - b^2\tau^2(\lambda_H h_L^2 + \lambda_L h_H^2)\lambda_L\lambda_H](1-\alpha)}{2(1+\beta)(1+\rho)[\lambda_L h_L^2 + \lambda_H h_H^2] - b\tau(h_L^2 + h_H^2)\lambda_L\lambda_H} \right]^{\frac{1+\alpha}{1-\alpha}} \tag{4.34}$$

4.3.3 Main Findings

As we have just seen, Eq. 4.27 displays the agents steady state labour supply under the unfunded NDC system. By assuming a dynamically efficient economy, where the return on PAYG NDC is lower than the market interest rate, high and low skilled workers tend to reduce the amount of labor supplied since they realize the missed opportunity of investing their contributions in stocks, bonds, or anything else. Conversely, Eq. 4.33 shows the agents steady state labor supply under the MFF system. If we take a step back, and thus if we set the individual fund's withdrawal b equal to zero we have the original version of a funded system. In a pure FF scheme, a variation in the contribution rate has no effect on individual labor supply irrespective of whether the worker is high or low skilled. This is a clear-cut result since in a pure FF individuals are aware that the payment of contributions into their own account is simply another form of private savings. Conversely, the introduction of a redistributive component in the funded pension ($b > 0$) creates labor supply distortions for both high and low skilled workers. In a MFF system, in fact, the former know that they will lose while the latter know that they will gain with respect to the pure FF. By looking at Eq. 4.29, we can see that the larger are the withdrawal b, the contribution rate τ, and the percentage of high skilled individuals in the population, the smaller is the incentive to work of low skilled agents.[15] Conversely, when the economy is mainly composed of low skilled individuals, high levels of both b and τ will create a disincentive to work for high skilled. Regarding the capital accumulation, which is expressed by Eqs. 4.26 and 4.32, we find that in both PAYG NDC and pure FF ($b = 0$) the level of capital per efficiency unit does not depend on the sizes of productive/less-productive groups. Conversely, as it will be clearer in the next section, in a MFF system with $b \neq 0$ the composition of the population matters since individuals adjust their choices over consumption and saving according to the utility they receive from redistribution.

Finally, both Propositions 1 and 2 support that the individual's labor supply at the steady state is higher in an aging economy (i.e., when $1 < \rho_{t+1} < 0$) with respect to the case in which the population is increasing from time t to time $t+1$ (i.e., when $\rho_{t+1} > 0$).

A deeper analysis of the changes in the individuals labor supply and saving decision under both PAYG NDC and MFF is provided in Section 4.4, where we complete the steady state analysis with a numerical investigation for realistic values of the parameters.

4.4 Numerical Results

The following subsections, namely Subsections 4.4.1 and 4.4.2, show for the different pension systems and for both high and low skilled workers, the nontrivial steady state values of labor supply choice and capital accumulation, expressed as functions of the parameter τ, for fixed values of the other parameters $\alpha, \beta, A, h_L, h_H, \lambda_L$ and ρ. Therefore, the equations we have to focus on are Eqs. 4.23-4.34. In particular, we choose the values

[15]The higher is the withdrawal from each individual accounts, the higher is the weight of the redistribution. For $b = 1$ we have a system where individuals receive the same benefit no matter of their productivity type.

$\alpha = 0.29$ and $A = 8$ following Bouzahzah et al. (2002). Moreover, we set $\beta = 0.96$, $h_L = 0.5$, $h_H = 1$, $\lambda_L = 0.3$, $\lambda_L = 0.7$ and $\rho = 0$.[16] Additionally, in Subsection 4.4.3 we investigate how the individual labor supply decision reacts to changes in ρ when the contribution rate is equal to 0.1 and 0.4, provided the other variables are set at the aforementioned values.

4.4.1 Labor Supply

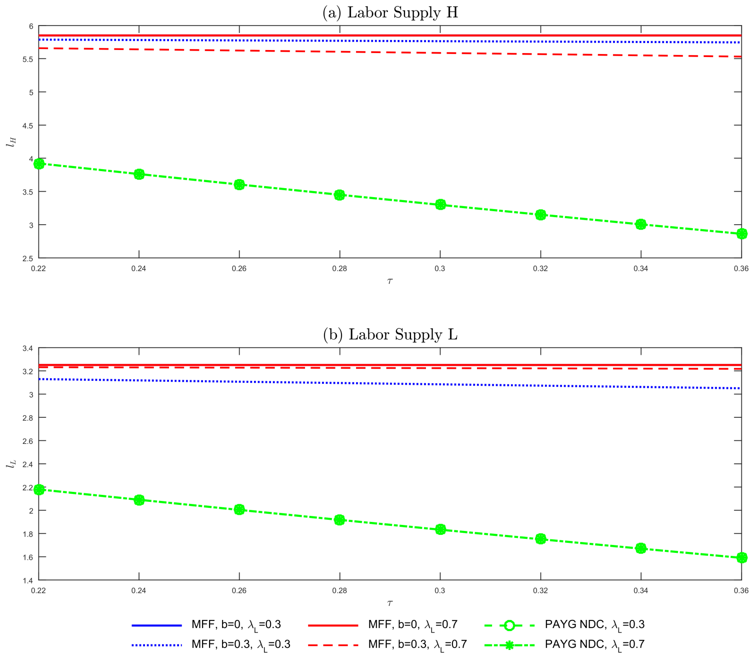
In a dynamic efficient economy with a PAYG NDC system, irrespective of the proportion of high and low skilled workers, an increase of the contribution rate τ will always decrease the labor supply of both high and low skilled workers (see Figure 8). Since the contributions paid by individuals during the entire working career are used to finance pensions of current period retirees, each workers claim for a pension is only based on the promise that future generations will be responsible for providing the benefit. Therefore, since the worker sees that the contributions he is paying now are revaluated at a lower interest rate than the market return, he will be more willing to supply less amount of labor.

In a pure FF system ($b = 0$), irrespective of the proportion of high and low skilled workers in the economy, the labor supply of high and low skilled workers is not related to the level of the contribution rate τ. The worker is fully aware that the contribution that he is paying during the working career is accumulated in a personal account to which he will have access at the time of retirement.

In order to illustrate the MFF system we chose for simplicity a redistribution component b equal to 0.3. From the graphs in Figure 8 it is easy to see that when the proportion of high skilled individuals prevails, i.e. when $\lambda_L = 0.3$ (blue dotted lines), while the labor supply distortion for high skilled workers is the smallest, the labor supply distortion for low skilled workers is the highest. When the composition of the population is the one just mentioned, high skilled individuals realize that the amount that they receive back with redistribution will not be much dif-

[16] We use $\lambda_L = 0.3$ to describe an economy mainly composed of high skilled workers. Conversely, we use $\lambda_L = 0.7$ to represent an economy where the majority of people belongs to the low skill class.

Figure 8: Labor supply of high and low skilled workers



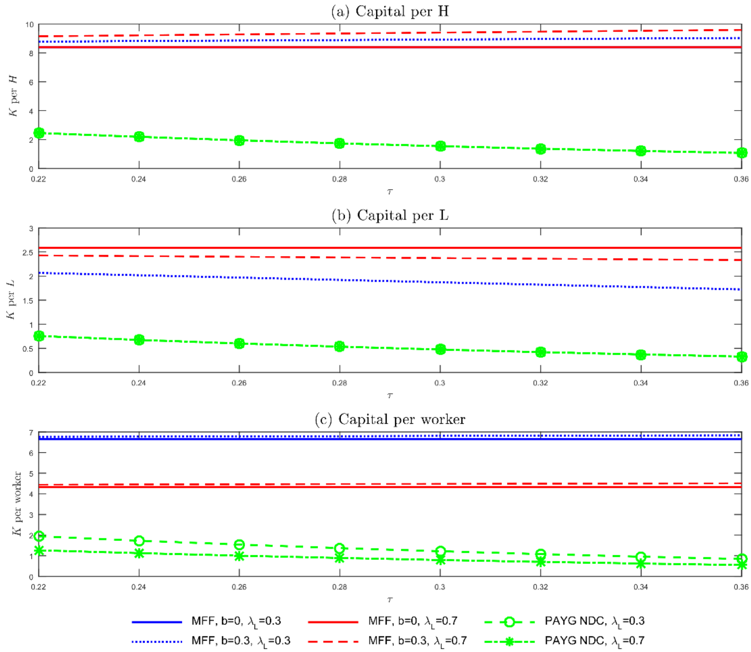
ferent from the fraction b that was previously taken from each individual account. For the same reason, low skilled workers are more inclined to work less since they know, due to the presence of a majority of high skilled individuals in the economy, that they will attain a higher than expected pension from redistribution.

4.4.2 Capital Accumulation

In order to understand the behavior of the steady state capital per worker $\frac{K_t}{N_t}$, which is illustrated in Figure 9 (c), we firstly present separately in Figure 9 (a) and (b) the capital per high skilled and low skilled individuals, respectively.

By looking at Figure 9 (a) and (b) we see that in a PAYG NDC system,

Figure 9: Capital Accumulation



when the contribution rate increases, it can be shown that the disposable income in the first period decreases and so do the savings (both comments follow from the equations presented in the Appendix B). Therefore, irrespective of the proportion of high and low skilled workers in the economy, both the capital per high skilled and low skilled worker tends to decrease. Obviously, for a given level of τ, the level of capital per worker for high skilled is higher than the one for low skilled.

Similarly, by looking at the first two graphs of Figure 9, we have that in a pure FF, for both high and low skilled workers, as τ increases the disposable income in the first period decreases and so do the savings. However, when the contribution rate increases, the resources accumulated in each worker's individual account rise. Overall, since the worker's future pension is generated by his personal first period savings and paid contribution, the capital per high and low skilled worker is independent on the level of τ. Needless to say that, for a specific level of the tax rate, the capital per worker for high skilled is higher than the one for low skilled.

In MFF, the analysis becomes more complex. In order to make the discussion as clear as possible we present the behavior of high skilled followed by the one of low skilled, i.e. we discuss Figure 9 (a) and then Figure 9 (b).

For high skilled workers it can be shown from the formulas used to prove Proposition 2 (see the Appendix B) that:

- First period savings are higher than in the case of pure FF ($b = 0$);
- First period savings are the highest when the proportion of low skilled individual prevails, i.e. when $\lambda_L = 0.7$;
- The amount of resources invested in the individual account is lower than in the pure FF ($b = 0$);
- The amount of resources invested in the individual account is the lowest when the proportion of low skilled individual prevails, i.e. when $\lambda_L = 0.7$.

Since high skilled workers know that a fraction b of their individual account will be withdrawn for redistribution purposes, as τ increases, they will be more prone to increase their personal savings in the first period, and to accumulate a lower amount of capital on their personal fund.[17] Actually, by expanding the savings in the first period and by reducing the accumulation of resources in their account, high skilled individuals preserve themselves from the large withdrawal associated to a richer fund. In a world where low skilled individuals are the majority, if high skilled individuals do not modify their choices on consumption and savings accordingly, as τ increases the amount that they get back once redistribution has occurred will be much lower than what they give in first place.

[17] More precisely, the reduction of both periods consumption experienced in MFF by high skilled workers is higher than in a pure FF case, and is the highest when the proportion of low skilled individual prevails, i.e. when $\lambda_L = 0.7$.

As can be seen from Figure 9 (a), as τ increases, in the MFF, the choices of high skilled workers are distorted with respect to the pure FF case: they save more and consume less. More precisely, the overall effect on the capital accumulation per high skilled worker is positive with respect to the original FF system.

Conversely, for low skilled workers it can be shown that:

- First period savings are lower than in the case of pure FF (when $b = 0$);
- First period savings are the lowest when the proportion of high skilled individual prevails, i.e. when $\lambda_L = 0.3$;
- The amount of resources invested in the individual account is the lowest when the proportion of high skilled individual prevails, i.e. when $\lambda_L = 0.3$.

One can easily see that since low skilled individuals know that there will be a redistribution, as τ increases, they will distort their choices over savings and consumption in both periods.[18] With respect to the pure FF case, as τ increases, if the high skilled group is the largest one, in addition to a reduction in the first period savings, low skilled workers tend to reduce the capital accumulation in their account. Conversely, when high skilled workers are the minority and τ rises, in addition to the decrease of first period savings, low skilled workers are likely to increase the resources accumulated in the personal fund (they know that the gains obtained from the redistribution are not so high).

Therefore, from Figure 9 (b) we see that, as τ increases, the choices of low skilled workers diverge from the pure FF case: they save less and consume more. In particular, in a MFF scheme the overall effect on the capital accumulation per low skilled worker is negative compared to the original FF.

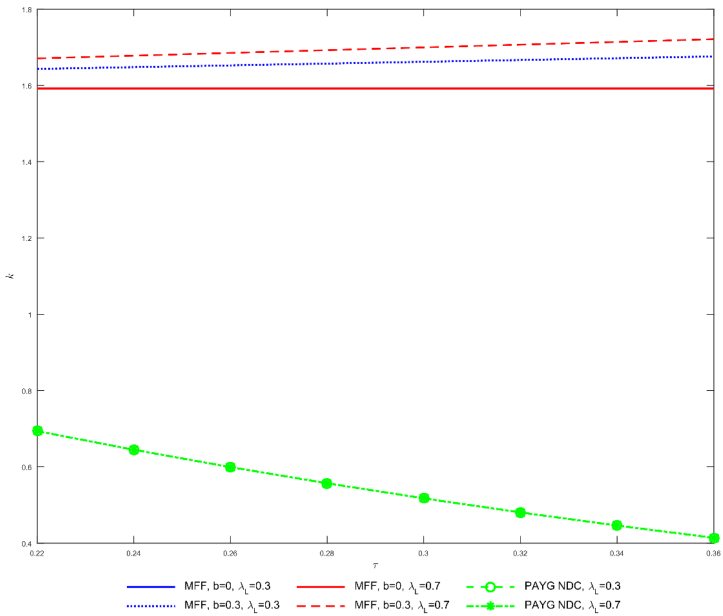
We can now move to the interpretation of Figure 9 (c). From the analysis conducted above, we have seen that while in a PAYG NDC a rise in

[18]It can be shown that in an MFF there is an increase of both periods consumption experienced by low skilled workers. Such increase is higher than in a pure FF, and is the highest when the proportion of high skilled individual prevails, i.e. when $\lambda_L = 0.3$.

the contribution rate τ depresses the level of capital per worker, in a pure FF system, the same variation in τ has no effect on capital accumulation. Conversely, in an MFF as τ increases, the rise in savings experienced by high skilled workers prevails on the reduction of the same driven by the behavior of low skilled individuals. Consequently, the introduction of a redistributive component in a pure funded system induces an increase in the accumulation of capital per worker (blue and red dotted lines are above the solid ones).

As regards the behavior of capital per efficiency unit under PAYG NDC, FF and MFF, we find the result reported in the following Figure 10.

Figure 10: Capital per efficiency unit



From Figure 10 we can infer that the capital per efficiency unit is the highest when the pension system is a MFF and when the population is largely composed by low skilled workers.

By recalling Eq. 4.9 and the results presented in Figure 8 above, we conclude that in the MFF scheme, provided the existence of a reduction in the labor supply by both groups of individuals (high and low skilled workers), labor supply contraction is higher when the proportion of low skilled individuals prevails. More precisely, the negative impact that a redistributive component has on the labor offer of high skilled workers is as much higher as the fraction of low skilled workers in the economy. Obviously, it follows that the lower the amount of labor supplied, the higher is k.

4.4.3 What happens when $\rho \neq 0$?

In this subsection, we remove the assumption of a non-growing population, i.e. we allow ρ to be different from zero. Keeping in mind the negative effect on labor supply induced by the presence of a tax on labor in both PAYG NDC and MFF, we now conduct a brief analysis on the effect of an increase in the tax rate τ when the population is either increasing either aging.

In Figure 11, we present the labor supply distortion suffered by high skilled workers as a result of an increase in the tax rate τ when even ρ is allowed to vary.[19] Moreover, since a variation in the tax rate does not affect the individual's behavior in a pure FF system, we present the PAYG NDC and the MFF cases only.

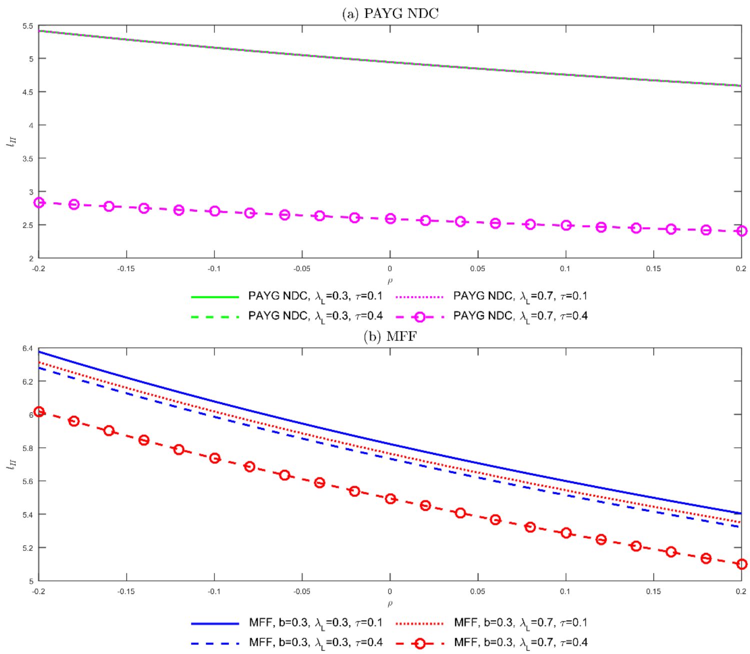
As one might expect, in a PAYG NDC system, irrespective on the proportion of high and low skilled workers in the economy, when taxes are higher, individuals are more willing to work less. Provided that, the amount of labor supplied by the single individual is higher in an aging economy. As ρ decreases in fact, the capital per efficiency unit increases and so does the worker's wage. From higher labor earnings, it follows a higher incentive to work.

When analyzing the possible reduction in labor supply coming from an increase in τ in a context where ρ is different from zero, we have

[19] The Appendix B shows that the behavior of low skilled workers is similar to the one of high skilled individuals.

to take into account that the composition of population matters. From the previous subsection, in fact we notice that the distortion of labor supply for high skilled workers is higher when the fraction of low skilled individuals is the largest. Other than that, we can follow the reasoning applied for the PAYG NDC case. Even in a MFF, as ρ decreases there exist a higher incentive to work.

Figure 11: Labor supply distortion when $\rho \neq 0$



4.5 Conclusions

Aging population and declining fertility rates require reforms that could help in creating more financially sustainable pension systems. The most credited solution, which consists in the privatization of social security, defined as the shift from unfunded Pay-as-you-go schemes to funded systems, does not come with disadvantages. In fact, while it allows overcoming the solvency problems of unfunded schemes, it is not able to pursue the relevant objectives of poverty relief and redistribution.

In a general equilibrium framework, this chapter studies labor market distortions and capital accumulation arising in different social security systems. Keeping in mind the solvency problem due to declining fertility and rising longevity, we restrict the analysis on three pension systems: the unfunded Pay-as-you-go Notional Defined Contribution, the Fully Funded, and the no more actuarially fair Fully Funded system, which is a modified funded scheme that includes an intragenerational redistributive component in order to carry on the redistribution purpose. According to our results, while the unfunded scheme depresses labor supply and physical capital accumulation, the original funded system discourages neither of the two. Furthermore, in the modified version of the funded system we propose, where the redistributive component takes the form of a withdrawal from each individual account, while high skilled workers consume less and increase their private savings in order to own a scarce account, low skilled individuals, who know they will gain from redistribution, increase consumption and reduce savings. Overall, MFF slightly increases physical capital accumulation with respect to the original funded system, without significantly reducing labor supply incentives. From a broader perspective, the introduction of MFF may help to reduce the burden of future intergenerational redistribution. Collecting and investing a share of each worker's contribution in a State-managed fund, and at the time of retirement redistributing such resources equally among the corresponding population, help to create a safety net for low-income pensioners contributing to alleviate the taxation burden on future cohorts of workers.

It is worthwhile to mention that, in order to reduce the distortionary effect on labor supply arising when funded systems embody an intragenerational redistributive component, it could be useful to introduce a minimum number of hours worked or a minimum level of effort in order for the individual to be eligible for redistribution.

Moreover, given the above discussion, even if the proposed MFF is able to reallocate resources among individuals, we are still in need of ensuring people against investment risk. If assets in which resources are invested perform poorly in fact, individuals face the risk of collecting inadequate pension benefits. In particular, in order to provide insurance against income uncertainty, the establishment of a proper regulation that imposes a banner on risky investments becomes of relevant importance. Actually, one can think, as it is happening in Chile, to create a Legal System that allows governments to provide transfers (up to a limit) to individuals who are not able to cover a minimum pension due, e.g., to poor investments performance.

Appendix B

B.1 Mathematical proofs

In this Appendix, we prove Propositions 1 and 2 in Subsections 4.3.1 and 4.3.2, deriving recursive formulas for the capital per efficiency unit for the different pension systems, and relating the capital per efficiency unit with individual labor supply choices. Starting from these recursive formulas, we also provide expressions for the nontrivial steady state values for the capital per efficiency unit, the individual labor supply choices, and for the capital per worker.

Proof of Proposition 1 **[PAYG NDC].**

Using Eq. 4.13, one obtains:

$$\begin{aligned} \Omega_{t+1}^{PAYG\,NDC} &= \frac{w_{t+1}^{PAYG\,NDC}\left(\lambda_L h_L l_{L,t+1}^{PAYG\,NDC} + \lambda_H h_H l_{H,t+1}^{PAYG\,NDC}\right)}{w_t^{PAYG\,NDC}\left(\lambda_L h_L l_{L,t}^{PAYG\,NDC} + \lambda_H h_H l_{H,t}^{PAYG\,NDC}\right)} \\ &= \frac{w_{t+1}^{PAYG\,NDC} L_{t+1}^{PAYG\,NDC}}{w_t^{PAYG\,NDC}(1+\rho_{t+1}) L_t^{PAYG\,NDC}}. \end{aligned} \tag{B.1}$$

this, combined with Eqs. 4.5-4.6, provides:

$$\begin{aligned}\frac{\Omega_{t+1}^{PAYG\ NDC}}{R_{t+1}^{PAYG\ NDC}} &= \frac{w_{t+1}^{PAYG\ NDC} L_{t+1}^{PAYG\ NDC}}{w_t^{PAYG\ NDC}(1+\rho_{t+1}) L_t^{PAYG\ NDC} R_{t+1}^{PAYG\ NDC}} \\ &= \frac{A(1-\alpha)\left(k_{t+1}^{PAYG\ NDC}\right)^{\alpha} L_{t+1}^{PAYG\ NDC}}{w_t^{PAYG\ NDC}(1+\rho_{t+1}) L_t^{PAYG\ NDC} A\alpha\left(k_{t+1}^{PAYG\ NDC}\right)^{\alpha-1}} \\ &= \frac{(1-\alpha) L_{t+1}^{PAYG\ NDC} k_{t+1}^{PAYG\ NDC}}{w_t^{PAYG\ NDC}(1+\rho_{t+1})\alpha L_t^{PAYG\ NDC}}.\end{aligned} \tag{B.2}$$

Using Eqs. 4.3 and 4.12, one obtains:

$$\begin{aligned}l_{i,t}^{PAYG\ NDC} &= (1-\tau) w_t^{PAYG\ NDC} h_i + \tau w_t^{PAYG\ NDC} h_i (1+\rho_{t+1}) \frac{\Omega_{t+1}^{PAYG\ NDC}}{R_{t+1}^{PAYG\ NDC}} \\ &= (1-\tau) w_t^{PAYG\ NDC} h_i + \tau w_t^{PAYG\ NDC} h_i (1+\rho_{t+1}) \\ &\cdot \frac{(1-\alpha) L_{t+1}^{PAYG\ NDC} k_{t+1}^{PAYG\ NDC}}{w_t^{PAYG\ NDC}(1+\rho_{t+1})\alpha L_t^{PAYG\ NDC}} \\ &= (1-\tau) w_t^{PAYG\ NDC} h_i + \frac{(1-\alpha)\tau h_i L_{t+1}^{PAYG\ NDC} k_{t+1}^{PAYG\ NDC}}{\alpha L_t^{PAYG\ NDC}} \\ &= (1-\tau) w_t^{PAYG\ NDC} h_i + \frac{(1-\alpha)\tau h_i k_{t+1}^{PAYG\ NDC}}{\alpha}(1+\rho_{t+1}) \\ &\cdot \frac{\lambda_L h_L l_{L,t+1}^{PAYG\ NDC} + \lambda_H h_H l_{H,t+1}^{PAYG\ NDC}}{\lambda_L h_L l_{L,t}^{PAYG\ NDC} + \lambda_H h_H l_{H,t}^{PAYG\ NDC}}.\end{aligned} \tag{B.3}$$

hence, when $i = L, H$, respectively:

$$\begin{aligned}l_{L,t}^{PAYG\ NDC} &= (1-\tau) w_t^{PAYG\ NDC} h_L + \frac{(1-\alpha)\tau h_L k_{t+1}^{PAYG\ NDC}}{\alpha}(1+\rho_{t+1}) \\ &\cdot \frac{\lambda_L h_L l_{L,t+1}^{PAYG\ NDC} + \lambda_H h_H l_{H,t+1}^{PAYG\ NDC}}{\lambda_L h_L l_{L,t}^{PAYG\ NDC} + \lambda_H h_H l_{H,t}^{PAYG\ NDC}}\end{aligned} \tag{B.4}$$

$$\begin{aligned}l_{H,t}^{PAYG\ NDC} &= (1-\tau) w_t^{PAYG\ NDC} h_H + \frac{(1-\alpha)\tau h_H k_{t+1}^{PAYG\ NDC}}{\alpha}(1+\rho_{t+1}) \\ &\cdot \frac{\lambda_L h_L l_{L,t+1}^{PAYG\ NDC} + \lambda_H h_H l_{H,t+1}^{PAYG\ NDC}}{\lambda_L h_L l_{L,t}^{PAYG\ NDC} + \lambda_H h_H l_{H,t}^{PAYG\ NDC}}\end{aligned} \tag{B.5}$$

Multiplying Eqs. B.4 and B.5 by h_H and h_L, respectively, and taking the difference, one obtains:

$$l_{L,t}^{PAYG\ NDC} h_H - l_{H,t}^{PAYG\ NDC}\ h_L = 0 \tag{B.6}$$

$$l_{H,t}^{PAYG\ NDC} = \tfrac{h_H}{h_L}\ l_{L,t}^{PAYG\ NDC} \tag{B.7}$$

$$l_{H,t+1}^{PAYG\ NDC} = \tfrac{h_H}{h_L} l_{L,t+1}^{PAYG\ NDC} \tag{B.8}$$

hence, Eq. B.4 simplifies to:

$$\begin{aligned} l_{L,t}^{PAYG\ NDC} &= (1-\tau)\, w_t^{PAYG\ NDC} h_L + \frac{(1-\alpha)\,\tau h_L k_{t+1}^{PAYG\ NDC}}{\alpha}(1+\rho_{t+1}) \\ &\cdot \frac{\lambda_L h_L l_{L,t+1}^{PAYG\ NDC} + \lambda_H h_H^2/h_L l_{L,t+1}^{PAYG\ NDC}}{\lambda_L h_L l_{L,t}^{PAYG\ NDC} + \lambda_H h_H^2/h_L l_{L,t}^{PAYG\ NDC}} \\ &= (1-\tau)\, w_t^{PAYG\ NDC} h_L + \frac{(1-\alpha)\,\tau h_L k_{t+1}^{PAYG\ NDC}}{\alpha} \cdot (1+\rho_{t+1})\, \frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}} \\ &= (1-\alpha)(1-\tau) A h_L \left(k_t^{PAYG\ NDC}\right)^{\alpha} + \frac{(1-\alpha)\,\tau h_L k_{t+1}^{PAYG\ NDC}}{\alpha} \\ &\cdot (1+\rho_{t+1})\, \frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}} \end{aligned} \tag{B.9}$$

Multiplying the two sides of Eq. B.9 by $l_{L,t}^{PAYG\ NDC}$:

$$\begin{aligned} &\left(l_{L,t}^{PAYG\ NDC}\right)^2 - (1-\alpha)(1-\tau) A h_L \left(k_t^{PAYG\ NDC}\right)^{\alpha} l_{L,t}^{PAYG\ NDC} \\ &- \frac{(1-\alpha)\,\tau h_L k_{t+1}^{PAYG\ NDC}}{\alpha} (1+\rho_{t+1})\, l_{L,t+1}^{PAYG\ NDC} = 0. \end{aligned} \tag{B.10}$$

this is a second-order algebraic equation with discriminant $\Delta \geq 0$, whose only non-negative solution is:

$$l_{L,t}^{PAYG\ NDC} = \frac{(1-\alpha)(1-\tau)Ah_L\left(k_t^{PAYG\ NDC}\right)^{\alpha}}{2} + \frac{\sqrt{(1-\alpha)^2(1-\tau)^2A^2h_L^2\left(k_t^{PAYG\ NDC}\right)^{2\alpha} + \frac{4\tau(1-\alpha)(1+\rho_{t+1})h_L}{\alpha}k_{t+1}^{PAYG\ NDC}\ l_{L,t+1}^{PAYG\ NDC}}}{2}. \tag{B.11}$$

In the following, we also determine another recurrence satisfied by the capital in efficiency units. To this aim, first we have to find expressions for the savings $s_{i,t}^{PAYG\ NDC}$. Using Eqs. B.2 and B.7, we can simplify the expression of $\frac{\Omega_{t+1}^{PAYG\ NDC}}{R_{t+1}^{PAYG\ NDC}}$ and determine the expression of $\frac{p_{i,t+1}^{PAYG\ NDC}}{R_{t+1}^{PAYG\ NDC}}$ as follows.

$$\begin{aligned}
\frac{\Omega_{t+1}^{PAYG\ NDC}}{R_{t+1}^{PAYG\ NDC}} &= \frac{w_{t+1}^{PAYG\ NDC}\left(\lambda_L h_L l_{L,t+1}^{PAYG\ NDC} + \lambda_H h_H l_{H,t+1}^{PAYG\ NDC}\right)}{w_t^{PAYG\ NDC}\left(\lambda_L h_L l_{L,t}^{PAYG\ NDC} + \lambda_H h_H l_{H,t}^{PAYG\ NDC}\right)A\alpha\left(k_{t+1}^{PAYG\ NDC}\right)^{\alpha-1}} \\
&= \frac{w_{t+1}^{PAYG\ NDC} l_{L,t+1}^{PAYG\ NDC}}{w_t^{PAYG\ NDC} l_{L,t}^{PAYG\ NDC} A\alpha\left(k_{t+1}^{PAYG\ NDC}\right)^{\alpha-1}} \\
&= \frac{A(1-\alpha)\left(k_{t+1}^{PAYG\ NDC}\right)^{\alpha} l_{L,t+1}^{PAYG\ NDC}}{A(1-\alpha)\left(k_t^{PAYG\ NDC}\right)^{\alpha} l_{L,t}^{PAYG\ NDC} A\alpha\left(k_{t+1}^{PAYG\ NDC}\right)^{\alpha-1}} \\
&= \frac{k_{t+1}^{PAYG\ NDC} l_{L,t+1}^{PAYG\ NDC}}{A\alpha\left(k_t^{PAYG\ NDC}\right)^{\alpha} l_{L,t}^{PAYG\ NDC}}.
\end{aligned} \tag{B.12}$$

$$\begin{aligned}
\frac{p_{i,t+1}^{PAYG\ NDC}}{R_{t+1}^{PAYG\ NDC}} &= \tau w_t^{PAYG\ NDC}(1+\rho_{t+1})h_i l_{i,t}\frac{\Omega_{t+1}^{PAYG\ NDC}}{R_{t+1}^{PAYG\ NDC}} \\
&= \tau w_t^{PAYG\ NDC}(1+\rho_{t+1})h_i l_{i,t}\frac{k_{t+1}^{PAYG\ NDC} l_{L,t+1}^{PAYG\ NDC}}{A\alpha\left(k_t^{PAYG\ NDC}\right)^{\alpha} l_{L,t}^{PAYG\ NDC}} \\
&= \tau A(1-\alpha)\left(k_t^{PAYG\ NDC}\right)^{\alpha}(1+\rho_{t+1})h_i l_{i,t}\frac{k_{t+1}^{PAYG\ NDC} l_{L,t+1}^{PAYG\ NDC}}{A\alpha\left(k_t^{PAYG\ NDC}\right)^{\alpha} l_{L,t}^{PAYG\ NDC}} \\
&= \tau(1-\alpha)(1+\rho_{t+1})h_i l_{i,t}\frac{k_{t+1}^{PAYG\ NDC} l_{L,t+1}^{PAYG\ NDC}}{\alpha l_{L,t}^{PAYG\ NDC}}.
\end{aligned} \tag{B.13}$$

hence, using Eqs. 4.2, B.7, and B.9, one gets the following expressions for $s_{L,t}^{PAYG\ NDC}$ and $s_{H,t}^{PAYG\ NDC}$:

$$\begin{aligned}
s_{L,t}^{PAYG\ NDC} &= \frac{\beta(1-\tau)^2 A^2 (1-\alpha)^2 \left(k_t^{PAYG\ NDC}\right)^{2\alpha} h_L^2}{1+\beta} \\
&+ \frac{\beta(1-\tau) A (1-\alpha)^2 \left(k_t^{PAYG\ NDC}\right)^{\alpha} h_L^2 \tau}{(1+\beta)} \frac{k_{t+1}^{PAYG\ NDC}}{\alpha} (1+\rho_{t+1}) \\
&\cdot \frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}} - \frac{\beta(1-\tau)^2 A^2 (1-\alpha)^2 \left(k_t^{PAYG\ NDC}\right)^{2\alpha} h_L^2}{2(1+\beta)} - \frac{\beta}{2(1+\beta)} \\
&\cdot \frac{(1-\alpha)^2 \tau^2 h_L^2 \left(k_{t+1}^{PAYG\ NDC}\right)^2}{\alpha^2} (1+\rho_{t+1})^2 \left(\frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}}\right)^2 \\
&- \frac{\beta(1-\tau) A (1-\alpha)^2 \left(k_t^{PAYG\ NDC}\right)^{\alpha} h_L^2 \tau}{(1+\beta)} \frac{k_{t+1}^{PAYG\ NDC}}{\alpha} (1+\rho_{t+1}) \\
&\cdot \frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}} - \frac{\tau(1-\alpha)(1+\rho_{t+1}) h_L l_{L,t}^{PAYG\ NDC}}{1+\beta} \frac{k_{t+1}^{PAYG\ NDC}}{\alpha} \frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}} \\
&= \frac{\beta(1-\tau)^2 A^2 (1-\alpha)^2 \left(k_t^{PAYG\ NDC}\right)^{2\alpha} h_L^2}{2(1+\beta)} - \frac{\beta}{2(1+\beta)} \\
&\cdot \frac{(1-\alpha)^2 \tau^2 h_L^2 \left(k_{t+1}^{PAYG\ NDC}\right)^2}{\alpha^2} (1+\rho_{t+1})^2 \left(\frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}}\right)^2 \\
&- \frac{\tau(1-\alpha)(1+\rho_{t+1}) h_L l_{L,t}^{PAYG\ NDC}}{1+\beta} \frac{k_{t+1}^{PAYG\ NDC}}{\alpha} \frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}}
\end{aligned} \tag{B.14}$$

$$\begin{aligned}
s_{H,t}^{PAYG\ NDC} &= \frac{\beta(1-\tau)^2 A^2 (1-\alpha)^2 \left(k_t^{PAYG\ NDC}\right)^{2\alpha} h_H^2}{2(1+\beta)} - \frac{\beta}{2(1+\beta)} \\
&\cdot \frac{(1-\alpha)^2 \tau^2 h_H^2 \left(k_{t+1}^{PAYG\ NDC}\right)^2}{\alpha^2} (1+\rho_{t+1})^2 \left(\frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}}\right)^2 \\
&- \frac{\tau(1-\alpha)(1+\rho_{t+1}) h_H^2 l_{L,t}^{PAYG\ NDC}}{h_L(1+\beta)} \frac{k_{t+1}^{PAYG\ NDC}}{\alpha} \frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}}.
\end{aligned} \tag{B.15}$$

Combining Eqs. B.14 and B.15, one gets:

$$
\begin{aligned}
K_{t+1}^{PAYG\ NDC} &= \sum_{i=L,H} N_t \lambda_i s_{i,t}^{PAYG\ NDC} \\
&= \frac{N_t}{1+\beta}\Bigg[\beta(1-\tau)^2 A^2 (1-\alpha)^2 \left(k_t^{PAYG\ NDC}\right)^{2\alpha} \frac{\lambda_L h_L^2 + \lambda_H h_H^2}{2} \\
&- \frac{\beta}{2}\frac{(1-\alpha)^2 \tau^2 \left(k_{t+1}^{PAYG\ NDC}\right)^2}{\alpha^2}(1+\rho_{t+1})^2 \left(\frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}}\right)^2 \left(\lambda_L h_L^2 + \lambda_H h_H^2\right) \\
&- \tau(1-\alpha)(1+\rho_{t+1})\frac{k_{t+1}^{PAYG\ NDC}}{\alpha} l_{L,t+1}^{PAYG\ NDC} \frac{\lambda_L h_L^2 + \lambda_H h_H^2}{h_L}\Bigg].
\end{aligned}
\tag{B.16}
$$

then, one obtains:

$$
\begin{aligned}
L_{t+1}^{PAYG\ NDC} &= N_{t+1}\left(\lambda_L h_L l_{L,t+1}^{PAYG\ NDC} + \lambda_H h_H l_{H,t+1}^{PAYG\ NDC}\right) \\
&= N_t\ (1+\rho_{t+1})\frac{\lambda_L h_L^2 + \lambda_H h_H^2}{h_L} l_{L,t+1}^{PAYG\ NDC}
\end{aligned}
\tag{B.17}
$$

$$
\begin{aligned}
k_{t+1}^{PAYG\ NDC} &= \frac{K_{t+1}^{PAYG\ NDC}}{L_{t+1}^{PAYG\ NDC}} \\
&= \frac{1}{2(1+\beta)(1+\rho_{t+1})\, l_{L,t+1}^{PAYG\ NDC}}\Bigg[A^2\beta(1-\alpha)^2(1-\tau)^2 h_L \\
&\cdot \left(k_t^{PAYG\ NDC}\right)^{2\alpha} - \frac{\beta\tau^2(1-\alpha)^2(1+\rho_{t+1})^2 h_L}{\alpha^2}\left(\frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}}\right)^2 \\
&\cdot \left(k_{t+1}^{PAYG\ NDC}\right)^2 - \frac{2\tau(1-\alpha)(1+\rho_{t+1})k_{t+1}^{PAYG\ NDC}}{\alpha} l_{L,t+1}^{PAYG\ NDC}\Bigg].
\end{aligned}
\tag{B.18}
$$

In the following, we also show how one can express $k_{t+1}^{PAYG\ NDC}$ as a function of $k_t^{PAYG\ NDC}$. Starting from Eq. B.10, one gets:

$$
l_{L,t+1}^{PAYG\ NDC} = \frac{l_{L,t}^{PAYG\ NDC}\left(l_{L,t}^{PAYG\ NDC} - (1-\alpha)(1-\tau)A h_L\left(k_t^{PAYG\ NDC}\right)^{\alpha}\right)}{\frac{(1-\alpha)\tau h_L k_{t+1}^{PAYG\ NDC}}{\alpha}(1+\rho_{t+1})}
\tag{B.19}
$$

which requires:

$$l_{L,t}^{PAYG\ NDC} \geq (1-\alpha)(1-\tau) A h_L \left(k_t^{PAYG\ NDC}\right)^{\alpha} \tag{B.20}$$

to guarantee the non-negativity of $l_{L,t+1}^{PAYG\ NDC}$. Then, using also Eq. B.18, one obtains:

$$\begin{aligned}
&2(1+\beta)(1+\rho_{t+1}) k_{t+1}^{PAYG\ NDC} \\
&\cdot \frac{l_{L,t}^{PAYG\ NDC}\left(l_{L,t}^{PAYG\ NDC} - (1-\alpha)(1-\tau) A h_L \left(k_t^{PAYG\ NDC}\right)^{\alpha}\right)}{\frac{(1-\alpha)\tau h_L k_{t+1}^{PAYG\ NDC}}{\alpha}(1+\rho_{t+1})} \\
&= A^2 \beta (1-\alpha)^2 (1-\tau)^2 h_L \left(k_t^{PAYG\ NDC}\right)^{2\alpha} - \frac{\beta \tau^2 (1-\alpha)^2 (1+\rho_{t+1})^2 h_L}{\alpha^2} \\
&\frac{\frac{\left(l_{L,t}^{PAYG\ NDC}\right)^2 \left(l_{L,t}^{PAYG\ NDC} - (1-\alpha)(1-\tau) A h_L \left(k_t^{PAYG\ NDC}\right)^{\alpha}\right)^2}{\frac{(1-\alpha)^2 \tau^2 h_L^2 \left(k_{t+1}^{PAYG\ NDC}\right)^2}{\alpha^2}(1+\rho_{t+1})^2}}{\left(l_{L,t}^{PAYG\ NDC}\right)^2} \left(k_{t+1}^{PAYG\ NDC}\right)^2 \\
&- \frac{2\tau(1-\alpha)(1+\rho_{t+1}) k_{t+1}^{PAYG\ NDC}}{\alpha} \\
&\cdot \frac{l_{L,t}^{PAYG\ NDC}\left(l_{L,t}^{PAYG\ NDC} - (1-\alpha)(1-\tau) A h_L \left(k_t^{PAYG\ NDC}\right)^{\alpha}\right)}{\frac{(1-\alpha)\tau h_L k_{t+1}^{PAYG\ NDC}}{\alpha}(1+\rho_{t+1})},
\end{aligned} \tag{B.21}$$

hence,

$$\begin{aligned}
&\frac{2\alpha(1+\beta) l_{L,t}^{PAYG\ NDC}\left(l_{L,t}^{PAYG\ NDC} - (1-\alpha)(1-\tau) A h_L \left(k_t^{PAYG\ NDC}\right)^{\alpha}\right)}{(1-\alpha)\tau h_L} \\
&= A^2 \beta (1-\alpha)^2 (1-\tau)^2 h_L \left(k_t^{PAYG\ NDC}\right)^{2\alpha} \\
&- \frac{\beta\left(l_{L,t}^{PAYG\ NDC} - (1-\alpha)(1-\tau) A h_L \left(k_t^{PAYG\ NDC}\right)^{\alpha}\right)^2}{h_L} \\
&- \frac{2\, l_{L,t}^{PAYG\ NDC}\left(l_{L,t}^{PAYG\ NDC} - (1-\alpha)(1-\tau) A h_L \left(k_t^{PAYG\ NDC}\right)^{\alpha}\right)}{h_L}.
\end{aligned} \tag{B.22}$$

After some simplifications, B.22 reduces to:

$$\frac{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}{(1-\alpha)\tau} = \beta\frac{(1-\alpha)(1-\tau)Ah_L\left(k_t^{PAYG\ NDC}\right)^{\alpha}}{l_{L,t}^{PAYG\ NDC}-(1-\alpha)(1-\tau)Ah_L\left(k_t^{PAYG\ NDC}\right)^{\alpha}}. \tag{B.23}$$

Concluding, one can express $l_{L,t}^{PAYG\ NDC}$ as a function of $k_t^{PAYG\ NDC}$ as follows:

$$l_{L,t}^{PAYG\ NDC} = \frac{[\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1-\alpha)(1-\tau)A\ h_L}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)} \cdot\left(k_t^{PAYG\ NDC}\right)^{\alpha} \tag{B.24}$$

which also satisfies Eq. B.20. Then, using Eq. B.7, one obtains:

$$l_{H,t}^{PAYG\ NDC} = \frac{[\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1-\alpha)(1-\tau)A\ h_H}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)} \cdot\left(k_t^{PAYG\ NDC}\right)^{\alpha}. \tag{B.25}$$

From Eq. B.24, one gets:

$$\frac{l_{L,t+1}^{PAYG\ NDC}}{l_{L,t}^{PAYG\ NDC}} = \frac{\left(k_{t+1}^{PAYG\ NDC}\right)^{\alpha}}{\left(k_t^{PAYG\ NDC}\right)^{\alpha}} \tag{B.26}$$

which combined with B.9 and B.24, provides:

$$\frac{[\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1-\alpha)(1-\tau)A\ h_L}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}\left(k_t^{PAYG\ NDC}\right)^{\alpha} = (1-\alpha)(1-\tau)Ah_L\left(k_t^{PAYG\ NDC}\right)^{\alpha} + \frac{(1-\alpha)\tau h_L k_{t+1}^{PAYG\ NDC}}{\alpha} \cdot(1+\rho_{t+1})\frac{\left(k_{t+1}^{PAYG\ NDC}\right)^{\alpha}}{\left(k_t^{PAYG\ NDC}\right)^{\alpha}}. \tag{B.27}$$

Then, after some simplifications, one obtains:

$$k_{t+1}^{PAYG\,NDC} = \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)(1+\rho_{t+1})} \right]^{\frac{1}{1+\alpha}} \left(k_t^{PAYG\,NDC}\right)^{\frac{2\alpha}{1+\alpha}}, \tag{B.28}$$

which is the desired recursive formula.

When $\rho_t = \rho$ for all t, the steady state expression of the capital in efficiency units is trivially obtained equating $k_{t+1}^{PAYG\,NDC}$ and $k_t^{PAYG\,NDC}$, whereas the steady state expressions for the individual labor supply choices and for the capital per person are obtained using B.24, B.7, and 4.9.

Summarizing the analysis above, for the PAYG NDC case one obtains the following.

Relationship between the individual labor supply choices and the capital per efficiency unit:

$$l_{i,t}^{PAYG\,NDC} = \frac{[\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1-\alpha)(1-\tau)A\,h_i}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)} \cdot \left(k_t^{PAYG\,NDC}\right)^{\alpha} \tag{B.29}$$

Relationship between the labor supply choices of both high and low skilled workers:

$$l_{H,t}^{PAYG\,NDC} = \frac{h_H}{h_L} l_{L,t}^{PAYG\,NDC}. \tag{B.30}$$

Recursive formula for the capital per efficiency unit:

$$k_{t+1}^{PAYG\,NDC} = \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho_{t+1})} \right]^{\frac{1}{1+\alpha}} \left(k_t^{PAYG\,NDC}\right)^{\frac{2\alpha}{1+\alpha}}. \tag{B.31}$$

Unique nontrivial steady state solution (when $\rho_t = \rho$ for all t):

$$k^{PAYG\,NDC} = \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)} \right]^{\frac{1}{1-\alpha}} \tag{B.32}$$

$$l_i^{PAYG\,NDC} = (1-\tau)A(1-\alpha)\frac{\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)} \cdot \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} \tag{B.33}$$

$$\frac{K_t^{PAYG\,NDC}}{N_t} = (\lambda_L h_L^2 + \lambda_H h_H^2)(1-\tau)A(1-\alpha) \cdot \frac{\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)} \cdot \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{1+\alpha}{1-\alpha}}. \tag{B.34}$$

***Proof of Proposition 2* [MFF].**

For simplicity, we start reporting the steps required for the case $\lambda_L = \lambda_H = \frac{1}{2}$. Combining Eqs. 4.3 and 4.17-4.20, one obtains:

$$\begin{aligned} l_{i,t}^{MFF} &= (1-\tau)w_t^{MFF}h_i + \frac{R_{t+1}^{MFF}\left((1-b)\tau w_t^{MFF}h_i + \frac{b\tau w_t^{MFF}h_i}{2}\right)}{R_{t+1}^{MFF}} \\ &= \left[(1-\tau)+\tau\left(1-\frac{b}{2}\right)\right]w_t^{MFF}h_i \\ &= \left(1-\frac{b\tau}{2}\right)w_t^{MFF}h_i, \end{aligned} \tag{B.35}$$

which implies:

$$l_{H,t}^{MFF} = \frac{h_H}{h_L}l_{L,t}^{MFF}. \tag{B.36}$$

Now, using Eq. B.35, we notice that:

$$\begin{aligned} &(1-b)\tau w_t^{MFF}h_L l_{L,t}^{MFF} + \sum_{i=L,H}\frac{b\tau w_t^{MFF}h_i l_{i,t}^{MFF}}{2} \\ &= \left(1-\frac{b}{2}\right)\tau w_t^{MFF}h_L\left(1-\frac{b\tau}{2}\right)w_t^{MFF}h_L + \frac{b}{2}\tau w_t^{MFF}h_H\left(1-\frac{b\tau}{2}\right)w_t^{MFF}h_H \\ &= \tau\left[\left(1-\frac{b}{2}\right)h_L^2 + \frac{b}{2}h_H^2\right]\left(1-\frac{b\tau}{2}\right)\left(w_t^{MFF}\right)^2, \end{aligned} \tag{B.37}$$

and similarly,

$$
\begin{aligned}
&(1-b)\,\tau w_t^{MFF} h_H l_{H,t}^{MFF} + \sum_{i=L,H} \frac{b\tau w_t^{MFF} h_i l_{i,t}^{MFF}}{2} \\
&= \tau\left[\left(1-\frac{b}{2}\right)h_H^2 + \frac{b}{2}h_L^2\right]\left(1-\frac{b\tau}{2}\right)\left(w_t^{MFF}\right)^2.
\end{aligned} \tag{B.38}
$$

These, combined with Eqs. 4.2 and 4.3, provide:

$$
\begin{aligned}
s_{L,t}^{MFF} &= \frac{\beta\,(1-\tau)\,w_t^{MFF} h_L l_{L,t}^{MFF} - \frac{\beta\left(l_{L,t}^{MFF}\right)^2}{2} - \frac{p_{L,t+1}^{MFF}}{R_{t+1}^{MFF}}}{1+\beta} \\
&= \frac{\beta\,(1-\tau)\left(1-\frac{b\tau}{2}\right)\left(w_t^{MFF}\right)^2 h_L^2}{1+\beta} - \frac{\beta\left(1-\frac{b\tau}{2}\right)^2\left(w_t^{MFF}\right)^2 h_L^2}{2\,(1+\beta)} \\
&- \frac{\tau\left(1-\frac{b}{2}\right)\left(1-\frac{b\tau}{2}\right)\left(w_t^{MFF}\right)^2 h_L^2}{1+\beta} - \frac{\tau\frac{b}{2}\left(1-\frac{b\tau}{2}\right)\left(w_t^{MFF}\right)^2 h_H^2}{1+\beta} \\
&= \frac{1-\frac{b\tau}{2}}{1+\beta}\left[\left(\frac{2\beta+b\tau\,(2+\beta)}{4} - \tau\,(1+\beta)\right)h_L^2 - \frac{b\tau}{2}h_H^2\right]\left(w_t^{MFF}\right)^2
\end{aligned} \tag{B.39}
$$

and

$$
s_{H,t}^{MFF} = \frac{1-\frac{b\tau}{2}}{1+\beta}\left[\left(\frac{2\beta+b\tau\,(2+\beta)}{4} - \tau\,(1+\beta)\right)h_H^2 - \frac{b\tau}{2}h_L^2\right]\left(w_t^{MFF}\right)^2. \tag{B.40}
$$

Then, using Eq. 4.22, and equations above, one gets:

$$
\begin{aligned}
K_{t+1}^{MFF} &= \frac{N_t}{2}\sum_{i=L,H}\left(s_{i,t}^{MFF} + (1-b)\tau w_t^{MFF} h_i l_{i,t}^{MFF} + b\tau w_t^{MFF} h_i l_{i,t}^{MFF}\right) \\
&= \left[\frac{(2+b\tau)\beta}{4} - \tau(1+\beta)\right]\frac{N_t(1-\frac{b\tau}{2})}{2(1+\beta)}(h_L^2+h_H^2)\,(w_t^{MFF})^2 \\
&+ \tau(1-b)\frac{N_t(1-\frac{b\tau}{2})}{2(1+\beta)}(h_L^2+h_H^2)(w_t^{MFF})^2 + b\tau\frac{N_t(1-\frac{b\tau}{2})}{2(1+\beta)}(h_L^2+h_H^2)(w_t^{MFF})^2 \\
&= \frac{N_t\beta(1-\frac{b\tau}{2})(1+\frac{b\tau}{2})}{4(1+\beta)}h_L^2 + h_H^2)(w_t^{MFF})^2,
\end{aligned} \tag{B.41}
$$

$$L_{t+1}^{MFF} = \frac{N_{t+1}}{2}\left(1-\frac{b\tau}{2}\right)w_{t+1}^{MFF}\left(h_L^2+h_H^2\right), \tag{B.42}$$

$$\begin{aligned}
k_{t+1}^{MFF} = \frac{K_{t+1}^{MFF}}{L_{t+1}^{MFF}} &= \frac{\frac{N_t\beta\left(1-\frac{b\tau}{2}\right)\left(1+\frac{b\tau}{2}\right)}{4(1+\beta)}\left(h_L^2+h_H^2\right)\left(w_t^{MFF}\right)^2}{\frac{N_{t+1}}{2}\left(1-\frac{b\tau}{2}\right)w_{t+1}^{MFF}\left(h_L^2+h_H^2\right)} \\
&= \frac{N_t\beta\left(1+\frac{b\tau}{2}\right)\left(w_t^{MFF}\right)^2}{2(1+\beta)(1+\rho_{t+1})w_{t+1}^{MFF}} \\
&= \frac{\beta\left(1+\frac{b\tau}{2}\right)A^2(1-\alpha)^2\left(k_t^{MFF}\right)^{2\alpha}}{2(1+\beta)(1+\rho_{t+1})A(1-\alpha)\left(k_{t+1}^{MFF}\right)^{\alpha}} \\
&= \frac{\beta\left(1+\frac{b\tau}{2}\right)A(1-\alpha)\left(k_t^{MFF}\right)^{2\alpha}}{2(1+\beta)(1+\rho_{t+1})\left(k_{t+1}^{MFF}\right)^{\alpha}},
\end{aligned} \tag{B.43}$$

which is the desired recursive formula.

When $\rho_t = \rho$ for all t, the steady state expression for the capital per efficiency unit is trivially obtained equating k_{t+1}^{MFF} and k_t^{MFF}, whereas the steady state expressions for the individual labor supply choices and for the capital per worker are obtained using B.35, 4.5, and 4.9.

Summarizing the analysis above, for the MFF case with $\lambda_L = \lambda_H = \frac{1}{2}$ one obtains the following:

Relationship between the individual labor supply choices and the capital per efficiency unit:

$$l_{i,t}^{MFF} = \left(1-\frac{b\tau}{2}\right)A(1-\alpha)h_i\left(k_t^{MFF}\right)^{\alpha}. \tag{B.44}$$

Relationship between the labor supply choices of both high and low skilled workers:

$$l_{H,t}^{MFF} = \frac{h_H}{h_L}l_{L,t}^{MFF}. \tag{B.45}$$

Recursive formula for the capital per efficiency unit:

$$k_{t+1}^{MFF} = \left[\frac{A\beta\left(1+\frac{b\tau}{2}\right)(1-\alpha)}{2(1+\beta)(1+\rho_{t+1})}\right]^{\frac{1}{1+\alpha}} \left(k_t^{MFF}\right)^{\frac{2\alpha}{1+\alpha}}. \tag{B.46}$$

Unique nontrivial steady state solution (when $\rho_t = \rho$ for all t):

$$k^{MFF} = \left[\frac{A\beta\left(1+\frac{b\tau}{2}\right)(1-\alpha)}{2(1+\beta)(1+\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{B.47}$$

$$l_i^{MFF} = \left(1-\frac{b\tau}{2}\right)A(1-\alpha)\left[\frac{A\left(1+\frac{b\tau}{2}\right)\beta(1-\alpha)}{2(1+\beta)(1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} \tag{B.48}$$

$$\frac{K_t^{MFF}}{N_t} = \left(1-\frac{b\tau}{2}\right)\frac{h_L^2+h_H^2}{2}A(1-\alpha)\left[\frac{A\left(1+\frac{b\tau}{2}\right)\beta(1-\alpha)}{2(1+\beta)(1+\rho)}\right]^{\frac{1+\alpha}{1-\alpha}}. \tag{B.49}$$

In the general case $\lambda_L \neq \lambda_H \neq \frac{1}{2}$, the formulas above are modified as follows:

$$l_{i,t}^{MFF} = (1-b\tau(1-\lambda_i))\,h_i. \tag{B.50}$$

$$l_{H,t}^{MFF} = \frac{(1-b\tau\lambda_L)\,h_H}{(1-b\tau\lambda_H)\,h_L}\,l_{L,t}^{MFF}. \tag{B.51}$$

$$\begin{aligned} K_{t+1}^{MFF} = &\frac{N_t[\lambda_L h_L^2+\lambda_H h_H^2-b^2\tau^2(\lambda_L h_L^2+\lambda_H h_H^2)\lambda_L\lambda_H]}{2(1+\beta)[\lambda_L h_L^2+\lambda_H h_H^2-b\tau(h_L^2+h_H^2)\lambda_L\lambda_H]} \\ &\cdot \lambda_L h_L^2+\lambda_H h_H^2-b\tau\lambda_L\lambda_H\left(h_L^2+h_H^2\right)\left(w_{t+1}^{MFF}\right)^2, \end{aligned} \tag{B.52}$$

$$L_{t+1}^{MFF} = N_{t+1}\left[\lambda_L h_L^2+\lambda_H h_H^2-b\tau\lambda_L\lambda_H\left(h_L^2+h_H^2\right)\right]w_{t+1}^{MFF}. \tag{B.53}$$

Summarizing, for the MFF case with $\lambda_L \neq \lambda_H \neq \frac{1}{2}$ one obtains the following.

Relationship between the individual labor supply choices and the capital per efficiency unit:

$$l_{i,t}^{MFF} = (1 - b\tau(1 - \lambda_i))\, A(1 - \alpha) h_i \left(k_t^{MFF}\right)^{\alpha}. \tag{B.54}$$

Relationship between the labor supply choices of both high and low skilled workers:

$$l_{H,t}^{MFF} = \frac{(1 - b\tau\lambda_L)\, h_H}{(1 - b\tau\lambda_H)\, h_L} l_{L,t}^{MFF}. \tag{B.55}$$

Recursive formula for the capital per efficiency unit:

$$k_{t+1}^{MFF} = \left[\frac{A\beta[\lambda_L h_L^2 + \lambda_H h_H^2 - b^2\tau^2(\lambda_H h_L^2 + \lambda_L h_H^2)\lambda_L\lambda_H](1-\alpha)}{2(1+\beta)(1+\rho_{t+1})[\lambda_L h_L^2 + \lambda_H h_H^2 - b\tau(h_L^2 + h_H^2)\lambda_L\lambda_H]}\right]^{\frac{1}{1+\alpha}} \cdot \left(k_t^{MFF}\right)^{\frac{2\alpha}{1+\alpha}}. \tag{B.56}$$

Unique nontrivial steady state solution (when $\rho_t = \rho$ for all t):

$$k^{MFF} = \left[\frac{A\beta[\lambda_L h_L^2 + \lambda_H h_H^2 - b^2\tau^2(\lambda_H h_L^2 + \lambda_L h_H^2)\lambda_L\lambda_H](1-\alpha)}{2(1+\beta)(1+\rho)[\lambda_L h_L^2 + \lambda_H h_H^2 - b\tau(h_L^2 + h_H^2)\lambda_L\lambda_H]}\right]^{\frac{1}{1-\alpha}}. \tag{B.57}$$

$$l_i^{MFF} = (1 - b\tau(1-\lambda_i))A(1-\alpha) \cdot \left[\frac{A\beta[\lambda_L h_L^2 + \lambda_H h_H^2 - b^2\tau^2(\lambda_H h_L^2 + \lambda_L h_H^2)\lambda_L\lambda_H](1-\alpha)}{2(1+\beta)(1+\rho)[\lambda_L h_L^2 + \lambda_H h_H^2 - b\tau(h_L^2 + h_H^2)\lambda_L\lambda_H]}\right]^{\frac{\alpha}{1-\alpha}} h_i. \tag{B.58}$$

$$\frac{K_t^{MFF}}{N_t} = [\lambda_L h_L^2 + \lambda_H h_H^2 - b\tau\lambda_L\lambda_H(h_L^2 + h_H^2)]A(1-\alpha) \cdot \left[\frac{A\beta[\lambda_L h_L^2 + \lambda_H h_H^2 - b^2\tau^2(\lambda_H h_L^2 + \lambda_L h_H^2)\lambda_L\lambda_H](1-\alpha)}{2(1+\beta)(1+\rho)[\lambda_L h_L^2 + \lambda_H h_H^2 - b\tau(h_L^2 + h_H^2)\lambda_L\lambda_H]}\right]^{\frac{\alpha}{1-\alpha}} h_i. \tag{B.59}$$

References

Aaron, H. (1966). The social insurance paradox. *Canadian Journal of Economics and Political Science/Revue canadienne de economiques et science politique*, 32(03):371–374. 3, 64

Auerbach, A. J., Gokhale, J., and Kotlikoff, L. J. (1991). Generational accounts-a meaningful alternative to deficit accounting. Technical report, National Bureau of Economic Research. 3, 10, 20

Auerbach, A. J., Gokhale, J., and Kotlikoff, L. J. (1992). Social security and medicare policy from the perspective of generational accounting. In *Tax Policy and the Economy, Volume 6*, pages 129–145. The MIT Press. 10, 20

Auerbach, A. J., Gokhale, J., and Kotlikoff, L. J. (1994). Generational accounting: a meaningful way to evaluate fiscal policy. *The Journal of Economic Perspectives*, 8(1):73–94. 3, 10, 20

Auerbach, A. J. and Oreopoulos, P. (2000). The fiscal effects of us immigration: a generational-accounting perspective. In *Tax Policy and the Economy, Volume 14*, pages 123–156. MIT Press. 10

Banca dItalia (2014). I bilanci delle famiglie italiane dell anno 2014. 27

Barr, N. and Diamond, P. (2006). The economics of pensions. *Oxford review of economic policy*, 22(1):15–39. 65

Benz, U. and Fetzer, S. (2006). Indicators for measuring fiscal sustainability: a comparison of the oecd method and generational accounting. *FinanzArchiv: Public Finance Analysis*, 62(3):367–391. 20, 23

Biffi, P. and Clemente, G. P. (2014). Selecting stochastic mortality models for the italian population. *Decisions in Economics and Finance*, 37(2):255–286. 58

Bisetti, E. and Favero, C. A. (2014). Measuring the impact of longevity risk on pension systems: The case of italy. *North American Actuarial Journal*, 18(1):87–103. 47

Blake, D. (2006). *Pension economics*. John Wiley & Sons. 65

Blake, D., Boardman, T., and Cairns, A. (2014). Sharing longevity risk: Why governments should issue longevity bonds. *North American Actuarial Journal*, 18(1):258–277. 62

Blake, D. and Burrows, W. (2001). Survivor bonds: Helping to hedge mortality risk. *Journal of Risk and Insurance*, pages 339–348. 62

Blanchard, O. J., Chouraqui, J.-C., Hagemann, R., and Sartor, N. (1991). The sustainability of fiscal policy: New answers to an old question. *NBER Working Paper*, (R1547). 10

Bonin, H. (2013). *Generational accounting: theory and application*. Springer Science & Business Media. 10, 20, 28

Bonin, H., Raffelhüschen, B., and Walliser, J. (2000). Can immigration alleviate the demographic burden? *FinanzArchiv: Public Finance Analysis*, 57(1):1–21. 10

Booth, H. and Tickle, L. (2008). Mortality modelling and forecasting: A review of methods. *Annals of actuarial science*, 3(1-2):3–43. 46

Borjas, G. J. (1994). The economics of immigration. *Journal of economic literature*, 32(4):1667–1717. 10

Borjas, G. J. and Bratsberg, B. (1994). Who leaves? the outmigration of the foreign-born. Technical report, National Bureau of Economic Research. 29

Bouzahzah, M., De la Croix, D., and Docquier, F. (2002). Policy reforms and growth in computable olg economies. *Journal of economic dynamics and control*, 26(12):2093–2113. 79

Breyer, F. and Straub, M. (1993). Welfare effects of unfunded pension systems when labor supply is endogenous. *Journal of Public Economics*, 50(1):77–91. 67

Cairns, A. J., Blake, D., and Dowd, K. (2006). A two-factor model for stochastic mortality with parameter uncertainty: Theory and calibration. *Journal of Risk and Insurance*, 73(4):687–718. 48, 58

Cardarelli, R. and Sartor, N. (2000). Generational accounting for italy. In *Fiscal Sustainability Conference*, page 501. 10

CNEL and MPLS (2012). CNEL - Consiglio Nazionale Economia e Lavoro e MPLS - Ministero del Lavoro e delle Politiche Sociali: Il ruolo degli immigrati nel mercato del lavoro italiano. 13

Coda Moscarola, F. (2001). The effects of immigration inflows on the sustainability of the italian welfare states. 10

Diamond, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review*, 55(5):1126–1150. 66, 67

Disney, R. (1996). *Can we afford to grow older?: a perspective on the economics of aging*. Mit Press. 65

Disney, R. et al. (1999). *Notional accounts as a pension reform strategy: An evaluation*. Social Protection, World Bank. 4, 64

Dustmann, C. (1996). Return migration: the european experience. *Economic policy*, 11(22):213–250. 29

Dustmann, C. (2003). Return migration, wage differentials, and the optimal migration duration. *European Economic Review*, 47(2):353–369. 29

EC (2015a). EC - European Commission: The 2015 Aging Report - economic and budgetary projections for the 28 EU Member States (2013-2060). http://ec.europa.eu/economy_finance/publications/. 2, 4, 25, 26, 27, 28, 34, 35, 47

EC (2015b). EC - European Commission: The 2015 Aging Report - Underlying Assumptions and Projection Methodologies. http://ec.europa.eu/economy_finance/publications/. 28

EC (2016). EC - European Commission: 2015 Fiscal sustainability report. http://ec.europa.eu/economy_finance/publications/. 1, 3, 4

EMN (2014). EMN - European Migration Network: Migrant access to social security and healthcare in italy, policies and practices. 14, 19

Eurostat (2013). Population projections europop2013. http://ec.europa.eu/eurostat/data/database. 28, 34, 47

Eurostat (2016a). Annual government finance statistics database - year 2013. http://ec.europa.eu/eurostat/data/database. 25, 26

Eurostat (2016b). Education and training database - year 2013. http://ec.europa.eu/eurostat/data/database. 27

Fetzer, S. (2006). *Zur nachhaltigen Finanzierung des gesetzlichen Gesundheitssystems*. Lang. 20

Fondazione Leone Moressa (2014). *Rapporto Annuale sull Economia dellImmigrazione La forza lavoro degli stranieri: esclusione o integrazione?* Il Mulino. 13, 15

Fornero, E. and Castellino, O. (2001). La riforma del sistema previdenziale italiano. *Il Mulino, Bologna*. 17

Franco, D., Gokhale, J., Guiso, L., Kotlikoff, L. J., Sartor, N., et al. (1992). *Generational accounting: the case of Italy*. Citeseer. 10

Hagist, C. (2008). *Demography and Social Health Insurance*. Nomos Verlagsgesellschaft mbH & Co. KG. 20, 31

Hagist, C., Moog, S., Raffelhüschen, B., Vatter, J., et al. (2009). Public debt and demography-an international comparison using generational accounting. Technical report. 10

Hagist, C., Raffelhüschen, B., Risa, A. E., Vardal, E., et al. (2011). Long-term fiscal effects of public pension reform in norway: A generational accounting analysis. *Research Center for Generational Contracts, University of Freiburg, Discussion Paper*, (49):36. 20

HMD (2015). HMD - Human Mortality Database. http://http://www.mortality.org. 46, 48, 50, 52, 53

Holzmann, R., Koettl, J., Chernetsky, T., et al. (2005). *Portability regimes of pension and health care benefits for international migrants: an analysis of issues and good practices*, volume 23. Global Commission on International Migration Geneva, Switzerland. 20

Holzmann, R., Palmer, E. E., et al. (2006). *Pension reform: Issues and prospects for non-financial defined contribution (NDC) schemes*. World Bank Publications. 4, 64

IDOS (2015). *IDOS - Centro Studi e Richerche: Dossier Statistico Immigrazione 2015*. Edizioni IDOS. 12, 13, 14, 15

Ignacio Conde-Ruiz, J. and Profeta, P. (2007). The redistributive design of social security systems. *The Economic Journal*, 117(520):686–712. 65

IMF (2012). IMF - International Monetary Fund: global financial stability report. http://www.imf.org/external/pubs/ft/gfsr/2012/01/pdf/text.pdf. 3, 46, 62

INPS (2014). INPS - Istituto Nazionale per la Previdenza Sociale: Osservatori statistici sugli extracomunitari e sui comunitari nati nei paesi dell'Europa dell'Est. Retribuzione dei lavoratori dipendenti. 15, 16, 27

INPS (2015). INPS - Istituto Nazionale per la Previdenza Sociale: L'INPS e le pensioni all'estero: un fenomeno in continua evoluzione. 19, 20

ISTAT (1998). ISTAT - Istituto Nazionale di Statistica: La presenza straniera in Italia negli anni 90. 11

ISTAT (2001). ISTAT - Istituto Nazionale di Statistica: 14 Censimento generale della popolazione e delle abitazioni. 12

ISTAT (2011). ISTAT - Istituto Nazionale di Statistica: 15 Censimento generale della popolazione e delle abitazioni. 12

ISTAT (2014a). ISTAT - Istituto Nazionale di Statistica: Condizioni di salute, fattori di rischio e ricorso ai servizi sanitari Anno 2013. 27

ISTAT (2014b). ISTAT - Istituto Nazionale di Statistica: Conti ed aggregati economici delle Amministrazioni Pubbliche anni 1995-2012. 25, 26

ISTAT (2014c). ISTAT - Istituto Nazionale di Statistica: I beneficiari delle prestazioni pensionistiche Anno 2011. 26

ISTAT (2015). ISTAT - Istituto Nazionale di Statistica: Rilevazione sulle forze di lavoro, IV Trimestre 2014, II Trimestre 2015. 13, 15, 16

Kotlikoff, L. J. and Raffelhüschen, B. (1999). Generational accounting around the globe. *The American Economic Review*, 89(2):161–166. 10

Lee, R. D. and Carter, L. R. (1992). Modeling and forecasting us mortality. *Journal of the American statistical association*, 87(419):659–671. 48, 51, 52

Moog, S., R. B., Reeker, G., et al. (2015). Ehrbare staaten? update 2015: Die nachhaltigkeit der ffentlichen finanzen in europa. *Argumente zur Marktwirtschaft und Politik*, (133). 35

MPLS (2014a). MPLS - Ministero del Lavoro e delle Politiche Sociali: la Comunitá Albanese in Italia, rapporto annuale sulla presenza degli immigrati. . 14, 15, 16, 27

MPLS (2014b). MPLS - Ministero del Lavoro e delle Politiche Sociali: la Comunitá Cinese in Italia, rapporto annuale sulla presenza degli immigrati. . 14, 15, 27

MPLS (2014c). MPLS - Ministero del Lavoro e delle Politiche Sociali: La Comunitá Marocchina in Italia, rapporto annuale sulla presenza degli immigrati. 14, 15, 27

MPLS (2014d). MPLS - Ministero del Lavoro e delle Politiche Sociali: La Comunitá Ucraina in Italia, rapporto annuale sulla presenza degli immigrati. 14, 15, 27

MPLS (2014e). MPLS - Ministero del Lavoro e delle Politiche Sociali: Quinto Rapporto Annuale, i migranti nel mercato del lavoro in Italia. 12, 13, 14, 15, 16, 27, 28

OECD (2014a). OECD - Organisation for Economic Co-operation and Development: Mortality assumptions and longevity risk, implications for pension funds and annuity providers. http://dx.doi.org/10.1787/9789264222748-en. 45, 48

OECD (2014b). OECD - Organisation for Economic Co-operation and Development: Pensions outlook 2014. http://dx.doi.org/10.1787/9789264222687-en. 2, 4, 5, 63, 65

OECD (2015). OECD - Organisation for Economic Co-operation and Development: Pensions at a glance 2015, OECD and G20 indicators. http://dx.doi.org/10.1787/pension_glance-2015-en. 1, 2, 4, 5, 17, 63

OECD (2016). OECD - Organisation for Economic Co-operation and Development: Statistics on health status. hhttps://stats.oecd.org/. 2, 45

OECD and EU (2015). OECD - Organisation for Economic Co-operation and Development and EU - European Union: Indicators of immigrant integration 2015. 13

Pitacco, E., Denuit, M., Haberman, S., and Olivieri, A. (2009). *Modelling longevity dynamics for pensions and annuity business*. OUP Oxford. 46

Raffelhüschen, B. (1999). Generational accounting in europe. *The American Economic Review*, 89(2):167–170. 10, 20, 23

Rizza, P. and Tommasino, P. (2010). Do we treat future generations fairly? italian fiscal policy through the prism of generational accounting. *Giornale degli Economisti e Annali di Economia*, pages 115–153. 10

Rowthorn, R. (2008). The fiscal impact of immigration on the advanced economies. *Oxford Review of Economic Policy*, 24(3):560–580. 10

Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *The journal of political economy*, pages 467–482. 3, 64

Samuelson, P. A. (1975). Optimum social security in a life-cycle growth model. *International Economic Review*, pages 539–544. 3, 64

Sinn, H.-W. (2000). Why a funded pension system is needed and why it is not needed. *International Tax and Public Finance*, 7(4-5):389–410. 65

Sommacal, A. (2006). Pension systems and intragenenerational redistribution when labor supply is endogenous. *Oxford Economic Papers*, 58(3):379–406. 67

Strozza, S. (2015). La presenza straniera in Italia. 16, 27

Wen, X., Pammolli, F., and Gnecco, G. (2015). Labor supply and capital accumulation in an aging economy: when beveridge meets bismarck. 70

WHIP (2015). Work Histories Italian Panel, banca dati di storie lavorative sviluppata grazie alla collaborazione tra Universitá di Torino e l'Inps. `http://www.laboratoriorevelli.it/whip`. 48, 49, 50

World Bank (2005). Notional accounts: notional defined contribution plans as a pension reform strategy. 4, 64, 65