Network Development and Regulation in Broadband Markets

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Abstract

This thesis analyses the issues related to broadband development, investment in Next Generation Networks (NGN) and regulation. The first chapter is a literature review on the determinants of adoption and usage of broadband, on one side, and on the determinants of the investment incentives in NGN and the impact of regulation, on the other side. The second and the third chapters are theoretical contributions to the literature on investment in NGN and regulation. We briefly describe the content of each chapter below.

First Chapter  This study surveys the theoretical literature and empirical evidence on two parallel and related issues regarding broadband development: the demand-side determinants of broadband adoption and usage and the relationship between investment incentives and regulation. The first section presents the results found in the literature on the main drivers of broadband adoption. We report also the empirical literature which attempts to disentangle “adoption” from “usage” of broadband. Further, the effect of location on adoption is examined, mainly regarding the rural/urban digital divide. Then, the interaction between fixed and mobile broadband is analysed, particularly discussing whether the adoption of fixed broadband and mobile broadband follows similar or diverging patterns. The second section reviews the literature regarding the determinants of the incentives to invest in broadband and their relationship with the regulatory framework. We also report the most recent studies on the main issues regarding the development of the Next Generation Networks (NGN).

Second Chapter  A vertically integrated incumbent and an OLO (Other Licensed Operator) compete in the market for broadband access. The incumbent has the option to invest in building a Next Generation Network that covers all urban areas with similar demand structures. The investment return in terms of demand increase is uncertain. We compare the impact of different access regulation regimes - full regulation, partial regulation (only the copper network is regulated), risk sharing - on investment incentives and social welfare. We find that, when the alternative for the OLO is using the copper network rather than leaving the market entirely, exclusion of the OLO does not necessarily
happen in equilibrium even when the incumbent is better in offering value-added services. Risk sharing emerges as the most preferable regime both from a consumer and a social welfare perspective for a large range of parameters.

**Third Chapter** We model the competition between an incumbent firm and an Other Licensed Operator (OLO) in the broadband market, where the incumbent has an investment option to build a Next Generation Network (NGN) and it can do so by making a risk sharing agreement with the OLO, or alone. Differently from other theoretical research, we discuss about two different kinds of risk sharing contractual forms – basic risk sharing, where no side-payment is given for the use of the NGN between co-investors, and joint-venture risk sharing, where a side-payment is set by the co-investing firms – and we compare them with the scenario in which the incumbent invests on a stand-alone basis. Then, we consider the introduction of a late entrant and we examine the related impact on the robustness of the risk sharing agreement and the equilibrium results. We find that risk sharing can potentially be beneficial in terms of competition and investments, but the number of firms involved matters and so does the choice of the NGN access price, for insiders and outsiders of the agreement. Although eventually the regulators’ objective is having no more network duplication and all operators using the NGN, it might not be an optimal strategy to start with all the firms in the market involved in a risk sharing agreement, unless the insiders NGN access charge is constrained at zero. Even when the presence of firms outside of the agreement force insiders to compete more fiercely, there might be a concern with the potential exclusion of the outsiders from the NGN. Therefore, a light regulation imposing no exclusion would be advisable.
Chapter 2

Technology Investment and Alternative Regulatory Regimes with Demand Uncertainty

2.1 Introduction

Telecommunications markets are experiencing a period of drastic technological development. The possibility to build a so-called Next Generation Network (NGN) gives firms the chance to exploit extremely faster transmission and thereby enrich their offer with more interactive and sophisticated services. However, the actual existence and importance of a demand for NGN applications is often uncertain\(^1\). The technology has been available for a while now, but given the high fixed costs needed to build the necessary infrastructure, and the risks due to demand uncertainty and regulatory uncertainty, the NGN deployment is progressing slowly in many countries.

The vexing issue as to how to provide firms with enough investment incentives, while eventually retaining the benefits of network development for final consumers, is highly debated by industry actors, regulators and scholars. In particular, access regulation is widely argued about its potential discouraging effect on regulated firms’ investment. When obliged to share its network elements with facilities-free rivals at a regulated access price, the incumbent may feel reluctant to invest in NGN because of the spillover effect enjoyed by the Other Licensed Operators (OLO). For these reasons, access regulation,

\(^1\)See for instance The Economist (2010) about lack of demand for NGN services in the United States.
mainly in the form of mandatory unbundling, may induce less or later incumbent’s investment compared to an unregulated scenario, but also compared to the socially desired level (Chang et al. (2003); Crandall and Singer (2003); Ingraham and Sidak (2003); Bourreau and Doğan (2005); Pindyck (2007); Grajek and Röller (2011), Nardotto et al. (2012)). The European Commission seems to acknowledge these concerns for future investments in NGN. In the Recommendation C(2010) 6223 on “Regulated Access to NGANs” (September 2010), the possibility of relaxing - if not eliminating - ex ante regulation when a risk sharing agreement backs up the deployment of NGN is openly considered.

The issue of broadband investment and regulation has attracted and still attracts a lot of research attention. Our paper contributes to this strand of literature by addressing the issue of access price setting when the incumbent has the option to invest in NGN and investment returns in terms of demand increase are uncertain. Using a model where a vertically integrated incumbent and an OLO dynamically compete in the market for broadband access, we analyse the effect of three different access regimes on the incentives to invest by the incumbent: full regulation (mandatory unbundling for NGN), partial regulation (no mandatory unbundling for NGN) and risk sharing. We then compare their impact on social welfare, balancing the effect of each regulatory regime on static and dynamic efficiency.

In our paper, we follow the original set-up of broadband investment and access regulation developed by Foros (2004). We develop a model with two firms having different ability to offer value-added services, and analyse the impact of access price regulation on the incumbent’s investment incentive. Differently from Foros (2004), however, we adopt a dynamic model of technology adoption and we include demand uncertainty for value-added NGN services. Considering that NGN investment might fail to expand market demand, we also assume that the OLO can possibly switch back to the copper network if there is no demand for NGN applications and the access to copper is cheaper. We then conduct our analysis comparing the impact on investment of three alternative access regimes. In this respect, the paper closer to ours is Nitsche and Wiethaus (2011). The authors analyse a simple two-stage framework with identical firms, where the incumbent is the only firm entitled with investment option and investment success in terms of demand increase is uncertain. Their work compares the impact of different modes of regulation (access price based on costs, risk sharing and regulatory holiday) in terms of investment extent and consumer welfare outcomes. There are several differences between our work and Nitsche and Wiethaus (2011)’s one. Firstly, in their model, following

2Cambini and Jiang (2009) provide a review of the theoretical and empirical literature on broadband investment and access regulation.
3A similar approach has been recently used by Mizuno and Yoshino (2012). In their model the authors analyse the incumbent’s incentive to invest under regulatory non-commitment, generalizing the results by Foros (2004). In our paper, instead, we use a dynamic investment model and demand uncertainty and we also compare different regulatory regimes in terms of their welfare implications. Our analysis is thus complementary to the Mizuno and Yoshino’s one.
Klumpp and Su (2010), the access charge is determined ex post from the equilibrium quantities and it includes a partial allocation of the fixed costs borne by the incumbent. In our model, the regulator establishes ex ante the level of access price, via first-order conditions. As a consequence, the benchmark case for access regulation in our model is a marginal cost-based rule, as in much of the literature in this field (Foros (2004), Kotakorpi (2006) for instance). Secondly, our setting is dynamic and we investigate the timing of investment in a context with demand uncertainty, rather than the extent of the investment. Moreover, we are able to carry out a complete welfare analysis, whereas Nitsche and Wiethaus (2011)’s work only gives an overview of the different modes of regulation’s implications in terms of consumer welfare. Lastly, our model includes quality differentiation à la Foros and considers its impact on equilibrium results, while, in Nitsche and Wiethaus (2011)’s model, firms are undifferentiated.

The impact of uncertainty on the timing of telecommunications infrastructure development has also been analysed in several papers that feature dynamic race models between incumbent and entrant operators and focus on specific access pricing regimes, mainly regulatory holidays (Hori and Mizuno (2006), Hori and Mizuno (2009), Gans (2001), Gans (2007) and Vareda and Homrig (2010)). In our model, by contrast, we consider uncertainty in a dynamic setting, but we focus on services-based competition, while taking into account different possible regulatory regimes.

Our paper also differs from a recent strand of studies that analyse an investment game where both the incumbent and entrants have the option to invest. Brito et al. (2012) examine the incentives of a vertically integrated firm (regulated at wholesale level) to invest and give access to a new (upgraded) wholesale technology, that is not subject to access regulation. Bourreau, Cambini and Doğan (2012) and Inderst and Peitz (2012a) analyse the incentives to migrate from an old technology to a new one, and examine how wholesale access conditions affect this migration. Finally, Manenti and Scialà (2011) study the impact of access regulation on entrant and incumbent’s investment and show that, in absence of regulation, the incumbent would choose an access charge to the new infrastructure that prevents resale-based entry, thereby overstimulating entrant’s investment which may eventually turn out to be inefficiently high.

Our model reveals that the differences in ability to provide value-added services and their absolute values with respect to the overall level of demand highly affect the investment choice. Since the OLO has the alternative to switch back to the copper network instead of leaving the market entirely, we find, in contrast to Foros (2004), that there are cases in which exclusion does not happen even when the incumbent is better in providing value-added services than the OLO. In case of mandatory switch to the NGN, we find that the OLO remains active in the market if and only if its ability to provide value-added services is higher than the incumbent’s one. The equilibrium results show that the
investment is always undertaken later than in the social optimum and that the presence of uncertainty has the effect of delaying the investment even further. Full regulation lowers the investment incentives due to a hold-up problem of the regulator who exploits the irreversibility of the incumbent’s investment ex post. Due to a combination of competitive intensity and investment incentives, we find that risk sharing is the most preferable regime from a consumer welfare perspective, but also from a total welfare perspective for a large range of parameters.

The remainder of the paper is organized as follows. Section 2 introduces the model and the main findings under the three different regulatory regimes. Section 3 summarises the paper and concludes.

2.2 The Model

We first present the basic features of the model. Then we present the results of the following regimes: full regulation, where the access charges to the legacy network and to the NGN are regulated; partial regulation, where only the access charge to the legacy network is regulated; risk sharing. Finally we illustrate the welfare comparisons between the different cases.

2.2.1 The Basic Framework

Two firms compete downstream for the provision of broadband connectivity. One firm is a vertically integrated incumbent, who owns the existing infrastructure, constituted by the copper network, and has the obligation to unbundle the network elements to its competitor under access regulation. The access fee to the existing infrastructure is assumed to be regulated at marginal cost. The second firm is a downstream competitor, leasing lines from the incumbent. Both firms provide the same services via the existing network, e.g. the conventional PC-centric services like www and email.

The incumbent firm has the option to invest in building a Next Generation Network (NGN). Such networks allow firms for a drastic improvement of the services provided, e.g. more speed in data transmission, enabling interactive TV-centric and gaming broadband services, IP-based and high definition TV, more capacity and faster connectivity.

The incumbent can decide at any time whether to invest in the NGN or to keep on using the copper network. Its investment choice is a one-time decision and it cannot be updated in a later period. Once it decides to invest, the incumbent must build a network that covers the entire market. When we talk about the entire market, we refer to regions that present roughly similar demand structures, particularly regarding the level of uncertainty about the NGN success. The rival can then decide whether to keep on using the copper network, or to ask the incumbent for access to the NGN by paying an access
fee. Alternatively, the incumbent and the entrant can jointly undertake and share the cost of the investment under a risk sharing agreement. In this case, we assume that each operator can use the NGN without any side payments.

Broadband services are sold by both operators to end-users at a fixed subscription fee independent of actual usage and time connected. Hence firms face downward sloping demand curves. Services provided by the two firms through the copper network are perfect substitutes. The adoption of NGN enriches the retail offer with value-added services. If the investment turns out to be successful, the chance to obtain value-added services increases consumers’ willingness to pay and shifts demand curves upwards for both firms. Consumers’ quality perception of the value-added services is differentiated between the two firms, so the respective market shares will be affected. In case of failure, there is no shift in demand.

We assume that there is Cournot competition in the retail market and the quantity sold is interpreted as the number of subscriptions. We assume that the access to the copper network is regulated at marginal cost level and there is no regulation in the retail market. Access pricing is the only regulatory tool in the context here, and, since the existing regulatory methods are designed for linear access pricing, we assume a linear access price. Furthermore, in line with the existing EU regulatory framework, we assume that the access charge to the new broadband network has to cover at least the network operating cost, so it cannot be set below marginal cost.4 Moreover, the regulator has an imperfect ability to make credible commitment, i.e. it is able to commit to a certain regulatory regime (full or partial regulation), but he cannot commit to the NGN access charge level before the investment is deployed.5

In order to reflect the need to foster the adoption of NGN in the current state of market uncertainty, we examine the conditions for which all operators decide to use the NGN as soon as it is deployed even though its success is still uncertain. Regulators want to make the switch to NGN faster, eventually avoiding the cost duplications arising from the overlapping use of copper network and NGN. Since the regulator cannot commit to a certain level of access charge before the investment is deployed, the decision upon the access conditions are taken after the investment stage. We assume that the regulator sets a sort of state-dependent access charge, as in Vareda (2010), that will adapt to the demand conditions once they become common knowledge with the phase of retail market competition.6

4As we will show in next paragraphs, this restriction, aside from being more realistic, is due to the OLO’s option to switch back to the “old” copper network. To make our analysis more complete, we will relax this assumption in section 2.2.4, forcing the OLO to switch to the NGN.
5Brito et al. (2010) consider how two-part tariffs can mitigate the regulatory commitment problem.
6State-dependent access rules are also adopted in Vareda (2010). The author analyses the impact of access regulation when an entrant has to decide whether to invest in a telecommunications network or to ask for access, and the regulator cannot observe its efficiency level. The paper shows that the regulator must set access prices contingent on demand, in order to induce the entrant to
In theory, the OLO can decide to use the NGN immediately after the investment is deployed, before the state of demand is revealed; or it can wait and see what is the true state of demand, and then decide which network to use. In order to limit the ex post entrant’s opportunism, i.e. the possibility that the entrant leaves the NGN if the realised demand happens to be (very) low, the regulator establishes access conditions under a long term contract, so that after the initial decision to switch to the new network, the OLO will continue to use the NGN. Therefore, we focus our analysis on access conditions for which the OLO switches to the NGN immediately after the investment is deployed and does not change its decision after the demand is revealed. Given the difficulties in the take-off of NGN networks in the current state of market uncertainty, indeed, analysing the circumstances under which more industry players would actually decide to initially join and use the NGN thereafter is a matter of greater social interest. In an extension of the model at section 2.2.4, we analyse the implications of a compulsory switch to the NGN for the OLO.

The timing of the model is the following (see Figure 2.1):^7

\[ \text{Decision on regulatory regime (partial or full regulation)} \]
\[ \text{Incumbent considers to invest in NGN, but does not do so} \]
\[ 0 \rightarrow t' \]
\[ t' \rightarrow t \]

\[ \text{Incumbent invests in NGN} \]
\[ \text{Period } t' \]

\[ \text{NGN created (with full regulation by regulator, with partial regulation by incumbent)} \]
\[ \text{Access charge is set} \]
\[ \text{OLO may decide on usage of network, or wait} \]
\[ \text{Demand realises} \]
\[ \text{Retail competition} \]

Figure 2.1: Timing of the Model

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^7 A similar structure of the game has been adopted by Mizuno and Yoshino (2012).
**Stage 0** At any time, the incumbent firm (together with the OLO, in case of risk sharing) decides whether to invest in building a NGN or to keep on using the copper network;

**Stage 1** Once the investment in the new network is deployed, with full access regulation (partial access regulation), the regulator (the incumbent firm) chooses the access price to be paid by the OLO to use the NGN;

**Stage 2**
- At any time after the access conditions have become common knowledge, the OLO decides whether to keep on using the legacy network or upgrade and ask access to the NGN;
- The state of demand is revealed and the two firms compete à la Cournot in the retail market.

Notice that in the risk sharing case, Stage 1 and the first bullet point of Stage 2 are absent.

**Demand Side**

Consumers have unit demand. Their valuation of a firm’s service is divided into two parts: one is for the basic broadband services and the other is for the value-added services running on NGN. Following Foros (2004), we assume the former is heterogeneous but the latter is homogeneous. Therefore a representative consumer’s valuation of firm \( i \)’s service is given by:

\[
\begin{align*}
    & \begin{cases} 
    v + \beta_i & \text{with probability } \gamma, \text{ case of success} \\
    v & \text{with probability } (1 - \gamma), \text{ case of failure}
    \end{cases}
\end{align*}
\]

Subscripts \( i = 1, 2 \) indicate incumbent and OLO, respectively. Here \( v \) is interpreted as the consumer’s willingness to pay for the basic service without new technology and is assumed to be uniformly distributed in \( (-\infty, a] \). Following Foros (2004), we allow for negative values of \( v \) in order to avoid corner solutions where all consumers enter the market. \( \beta_i \) describes firm \( i \)’s ability to offer value-added services after a successful investment and is assumed to belong to the interval \( (0, g) \) with \( g = a - c > 0 \), where \( c \) is the marginal cost for the provision of value-added services. Unlike Nitsche and Wiethaus (2011) and similarly to Foros (2004), firms’ abilities are differentiated. Notice also that there is no chance here for an overall “drastic” or “non-drastic” investment, as in Brito et al. (2012), since the market is never covered.\(^8\) As in Nitsche and Wiethaus (2011), market success is uncertain: the investment increases consumers’ willingness to pay by \( \beta_i \) with a probability equal to \( \gamma \), with \( \gamma \in (0, 1) \); or, consumers’ willingness to pay does not increase at all, even though NGN

\(^8\)Brito et al. (2012) use a Hotelling framework for the demand, which implies the possibility of all consumers preferring one firm to the other.
enhances the quality of services, with a probability equal to \((1 - \gamma)\).

The subscription fee charged by firm \(i\) is \(p_i\). A representative consumer buys from firm \(i\) other than firm \(j\) (\(j = 1, 2\) and \(j \neq i\)) if the following conditions are satisfied:

\[
\begin{align*}
\{ v + \beta_i - p_i > v + \beta_j - p_j \} & \quad \text{with probability } \gamma, \text{ case of success} \\
\{ v - p_i > v - p_j \} & \quad \text{with probability } (1 - \gamma), \text{ case of failure}
\end{align*}
\]

Therefore the firms’ quality-adjusted prices \(P\) should be equal if both firms are active in the market:

\[
\begin{align*}
\{ p_i - \beta_i = p_j - \beta_j = P \} & \quad \text{with probability } \gamma, \text{ case of success} \\
\{ p_i = p_j = P \} & \quad \text{with probability } (1 - \gamma), \text{ case of failure}
\end{align*}
\]

Consumers whose willingness to pay for the basic service \(v\) is no lower than the quality-adjusted price \(P\) enter the market, so there are \(a - P\) active consumers. The total quantity provided by firms is \(Q = q_1 + q_2\), so we have \(Q = a - P\). Thus the inverse demand functions faced by the firms are:

- **case of success**
  \[
  \begin{align*}
  p_1^s &= a + \beta_1 - q_1^s - q_2^s \\
  p_2^s &= a + \beta_2 - q_1^s - q_2^s 
  \end{align*}
  \]

- **case of failure**
  \[
  \begin{align*}
  p_1^f &= a - q_1^f - q_2^f \\
  p_2^f &= a - q_1^f - q_2^f 
  \end{align*}
  \]

With the superscripts \(s\) and \(f\) we denote the case of investment’s success and failure, respectively. Note that \(p_i^s\) here is a quality-adjusted Cournot price, which captures firm \(i\)’s ability to provide value-added services. Since such abilities are differentiated between the two firms, the quality-adjusted prices differ between the incumbent and the OLO, in case of success. The demand for basic services running on the copper network, \(p_i^C\) is the same as the demand in case of failure, so we have that

\[
\begin{align*}
\{ p_1^C = a - q_1^C - q_2^C \\
\{ p_2^C = a - q_1^C - q_2^C 
\end{align*}
\]

**Supply side**

A local connection to an end user is composed of two main elements, namely, a local line and a line card. The first cost is borne by the network owner for maintaining the daily operation of the essential input and is normalised to 0 in
our model without loss of generality. The second cost, incurred to provide services to end users at retail level, is assumed to be constant and equal to $c > 0$. We assume that a market for the broadband access service exists, i.e. $a > c$.

The access charge to the copper network and to the NGN are denoted with $r^C$ and $r^l$, respectively, where the superscript $l = P, F$ corresponds to the cases of partial regulation and full regulation, respectively. The level of access charge is decided by the incumbent, in case of partial regulation, or by the regulator, in case of full regulation.

We assume that the regulator sets the access charge after the investment is deployed, being aware of the presence of demand uncertainty. Hence, the access charge to the NGN becomes $r^l$ in case of failure and $r^{ls}$ in case of success.

9 We have also solved the case where the regulator sets a single access charge for the NGN, independent of demand. We discuss the solution of this case in footnote ??.

The investment in NGN entails a quadratic adoption cost given by $C_i(m, \Delta) = m^2 \Delta^2 \phi / 2$. $\Delta \in [0, 1]$ is the discount factor determined by the new-technology adoption date. Here we use the same notation and interpretation as in Bourreau and Doğan (2005) that $\Delta = \exp(-\delta t)$ where $\delta$ is the discount rate normalised to 1 and $t$ denotes time. $\Delta$ reflects the investment timing: a higher $\Delta$ corresponds to an earlier investment. The extent of network updating is represented by $m \in [0, 1]$. In our setting, the incumbent chooses $\Delta$ optimally and invests in the whole network, i.e. $m = 1$, so $C_i(\Delta) = \Delta^2 \phi / 2$. $\phi$ is a positive cost parameter. We assume the following: $\frac{d}{d\Delta} C \geq 0$ and $\frac{d^2}{d\Delta^2} C > 0$. Notice that since the investment cost decreases with time, there is no case in which there is no investment in this setting, unlike in Brito et al. (2012).

The ex ante profits of the two firms are the following:

$$
\begin{align*}
\pi_1^C &= (1 - \Delta) \pi_{1C}^C + \Delta (\gamma \pi_{1s}^C + (1 - \gamma) \pi_{1f}^C) \\
\pi_2^C &= (1 - \Delta) \pi_{2C}^C + \Delta (\gamma \pi_{2s}^C + (1 - \gamma) \pi_{2f}^C)
\end{align*}
$$

Here, firms’ profits before the investment, denoted by the superscript $C$ in the equations above to represent the use of the copper network, are equal to:

$$
\begin{align*}
\pi_{1C}^C &= (p_{1C}^C - c) q_{1C}^C + r^C q_{1C}^C \\
\pi_{2C}^C &= (p_{2C}^C - c) q_{2C}^C - r^C q_{2C}^C
\end{align*}
$$

Firms’ profits after investing in NGN, provided that the OLO also decides to use the new infrastructure, are different depending on the true state of demand.

- case of success

$$
\begin{align*}
\pi_{1s}^s &= (p_{1s}^s - c) q_{1s}^s + r^{ls} q_{1s}^s - \alpha^l \Delta \phi / 2 \\
\pi_{2s}^s &= (p_{2s}^s - c) q_{2s}^s - r^{ls} q_{2s}^s - (1 - \alpha^l) \Delta \phi / 2
\end{align*}
$$
case of failure:

\[
\begin{align*}
\pi_{1}^{f} &= (p_{1}^{f} - c)q_{1}^{f} + r_{1}^{f} q_{2}^{f} - \alpha^f \Delta \phi / 2 \\
\pi_{2}^{f} &= (p_{2}^{f} - c)q_{2}^{f} - r_{1}^{f} q_{2}^{f} - (1 - \alpha^f) \Delta \phi / 2
\end{align*}
\]

The parameter \( \alpha \in [0, 1] \) represents the way in which the investment cost is shared between the two firms. So we have that \( \alpha^P = \alpha^F = 1 \) and \( \alpha^{RS} \in (0, 1) \), because in case of partial regulation or full regulation the investment is undertaken by the incumbent alone, while in case of risk sharing the investment cost is somehow split among the two firms.

The following assumption is made for the model.

**Assumption 2.1.** \( r^F \geq 0 \) and \( r^C = 0 \)

This constraint imposes a lower bound limit to the NGN access price set by the regulator, \( r^F \), which cannot be lower than the network operation marginal cost, as in Foros (2004). In other words, the incumbent must have a non-negative price cost margin on its sale to the OLO if the NGN access market is regulated. In the second part of Assumption 3.1, we assume that the access fee to the copper network, \( r^C \), is regulated at marginal cost level. This second assumption restricts our attention to the problem of access price setting in the NGN market and reflects the current situation in many countries, where the regulation of the legacy network access charge is strictly cost-based.

**Social Welfare**

The social welfare function faced by the regulator at the moment of the access fee setting is composed of a pre-investment part and a post-investment part, with \( l = P, F, RS \), in the following way:

\[
E(W) = (1 - \Delta) W^C + \Delta E(W^l)
\]

with

\[
W^C = \left( \frac{a - p_{1}^{C}}{2} q_{1}^{C} + \frac{a - p_{2}^{C}}{2} q_{2}^{C} + \pi_{1}^{C} + \pi_{2}^{C} \right)
\]

\[
E(W^{N,l}) = \gamma \left( \frac{a + \beta_{1} - p_{1}^{l,s}}{2} q_{1}^{l,s} + \frac{a + \beta_{2} - p_{2}^{l,s}}{2} q_{2}^{l,s} + \pi_{1}^{l,s} + \pi_{2}^{l,s} - \Delta \phi / 2 \right) + (1 - \gamma) \left( \frac{a - p_{1}^{l,f}}{2} q_{1}^{l,f} + \frac{a - p_{2}^{l,f}}{2} q_{2}^{l,f} + \pi_{1}^{l,f} + \pi_{2}^{l,f} - \Delta \phi / 2 \right)
\]
Stage 2: Retail Market Competition

Firms compete under Cournot competition in the retail market. The resulting equilibrium quantities in this segment are:

- Before investment
  \[ q_{1}^{C*} = \frac{a - c}{3}, \quad q_{2}^{C*} = \frac{a - c}{3} \]

- After successful investment
  \[ q_{1}^{ls*} = \frac{a - c + r^{ls} + 2\beta_1 - \beta_2}{3}, \quad q_{2}^{ls*} = \frac{a - c - 2r^{ls} + 2\beta_2 - \beta_1}{3} \]

- After unsuccessful investment
  \[ q_{1}^{lf*} = \frac{a - c + r^{lf}}{3}, \quad q_{2}^{lf*} = \frac{a - c - 2r^{lf}}{3} \]

with \( l = P, F, RS \) denoting the different regulatory regimes.

We now make the following assumption.

**Assumption 2.2.** \( 2\beta_i \geq \beta_j, \forall i, j = 1, 2 \) with \( i \neq j \)

The above inequality implies that the difference in ability to provide value-added services between firms is not too large. Therefore with any given access price \( r^l \), each firm’s quantity is a non decreasing function with respect to the investment. Under this assumption, the incumbent cannot use the investment in NGN as a foreclosure tool (Foros (2004)).

**Stage 2: the OLO chooses whether to use the NGN**

*Ex ante*, the OLO decides to ask access to the NGN only if the expected profits from doing so are not lower than the profits obtainable by providing services through the copper network, whose access price is regulated at marginal cost level:

\[ E(\pi_{2}^l) = \gamma \pi_{2}^{ls} + (1 - \gamma) \pi_{2}^{lf} \geq \pi_{2}^{C} \]

with \( l = P, F, RS \).

Once we insert the equilibrium quantities, this inequality implies that:

\[ \gamma \left( \frac{a - c - 2r^{ls} + 2\beta_2 - \beta_1}{3} \right)^2 + (1 - \gamma) \left( \frac{a - c - 2r^{lf}}{3} \right)^2 \geq \left( \frac{a - c}{3} \right)^2 \quad (2.1) \]

If the above condition is satisfied, the OLO will switch to the NGN once the investment is deployed, but its success is still uncertain. In this case, the OLO does not behave opportunistically and use the NGN even in case of failure.
Notice that the OLO would be indifferent to the relative balancing of the access charges in case of failure and success, as long as the expected value of profits respects Condition 2.1, but we restrict our analysis to the case where using the NGN gives at least as much profit as the copper network to the OLO. We impose this restriction to focus on the most plausible equilibrium and to make the case of partial regulation readily comparable to the case of full regulation. Following this restriction, in case of failure it must be true that:

$$\left(\frac{a - c - 2r_{lf}}{3}\right)^2 \geq \left(\frac{a - c}{3}\right)^2$$

(2.2)

where the left hand side of the equation is the profit with the NGN and the right hand side is the profit with copper network.

Following from the assumption that the copper network access price is regulated at marginal cost level, Condition 2.2 requires that:

$$r_{lf} = r_C = 0$$

(2.3)

Therefore, the access fee in case of failure will respect Condition (2.3) and profits will be the same as with the copper network under all regulatory regimes:

$$\pi_{lf}^1 = \left(\frac{a - c}{3}\right)^2, \pi_{lf}^2 = \left(\frac{a - c}{3}\right)^2$$

After substituting the expression for $$\pi_{lf}^2$$, we can simplify the OLO’s ex ante constraint 2.1 in the following way:

$$\left(\frac{a - c - 2r_{ls} + 2\beta_2 - \beta_1}{3}\right)^2 \geq \left(\frac{a - c}{3}\right)^2$$

(2.4)

### 2.2.2 Full Regulation

We consider this case as a benchmark for cost-based regulation, where the regulator chooses the access charge by maximising a standard welfare function. In our case, cost-based regulation translates in marginal cost pricing, so the regulator only ensures to cover the incumbent’s operating costs.

*Stage 1: the regulator sets the access price to the NGN*

In this case, the regulator sets the access rule to the NGN in order to max-
imise social welfare. Its objective function after investment is the following:

\[ E(W^{NF}) = \gamma \left( \frac{(q_1^{F*} + q_2^{F*})^2}{2} + (q_1^{F*})^2 + r^{F*} q_2^{F*} - \Delta \phi/2 + (q_2^{F*})^2 \right) \]

\[ (1 - \gamma) \left( \frac{(q_1^{Ff*} + q_2^{Ff*})^2}{2} + (q_1^{Ff*})^2 - \Delta \phi/2 + (q_2^{Ff*})^2 \right) \]

We remind that \( r^{Ff} = 0 \) by Condition 2.3.\(^{10} \) The first-order condition with respect to \( r^{Fs} \) gives the access price as:

\[ r^{Fs*} = c - a + 4\beta_1 - 5\beta_2 \]

\( c - a < 0 \) is a necessary condition for a broadband market to exist. If \( \beta_1 > \beta_2 \) so much that \( 4\beta_1 - 5\beta_2 > a - c \), then the solution to the first-order condition given by the expression above is positive, \( r^{Fs*} > 0 \), implying that the regulator sets an above cost access charge.

If, otherwise, the incumbent is worse than the OLO in offering value-added services, \( \beta_1 \leq \beta_2 \), or if it is better in offering value-added services but not by a great extent, \( \beta_1 > \beta_2 \) but \( 4\beta_1 - 5\beta_2 < a - c \), the solution to the first-order condition is lower than the incumbent’s marginal cost of network operations, i.e. \( r^{Fs*} < 0 \). The regulator, indeed, not only values the fact that the OLO is able to increase demand through \( \beta_2 \), as also the incumbent does through \( \beta_1 \), but it also values that the OLO’s presence increases competition downstream. This is the reason why, in order to encourage the OLO’s participation into the NGN market, the regulator may set a below-cost access charge. However, \( r^{Fs*} < 0 \) contradicts Assumption 3.1, according to which \( r^{Fs*} \geq 0 \), so in this case we will impose \( r^{Fs*} = 0 \), such that optimal regulated access price will be set equal to the marginal cost.

The access price in case of full regulation is as following:

\[ r^{Fs} = \begin{cases} 
0 & \text{if } 4\beta_1 - 5\beta_2 \leq a - c \\
(c - a + 4\beta_1 - 5\beta_2) & \text{otherwise}
\end{cases} \]

By substituting the values for \( r^{Fs} \) into the expressions for the equilibrium quantities, we obtain the following expected quantities:

\[ E(q_1^{F*}) = \begin{cases} 
\gamma \left( \frac{a-c+2\beta_1-\beta_2}{4} \right) + (1 - \gamma) \left( \frac{a-c}{4} \right) & \text{if } 4\beta_1 - 5\beta_2 \leq a - c \\
\gamma 2(\beta_1 - \beta_2) + (1 - \gamma) \left( \frac{a-c}{4} \right) & \text{otherwise}
\end{cases} \]

\(^{10} \)In case of failure, the regulated access charge is set at the marginal cost level. From a policy point of view, it would be more suitable and less distortive to use other instruments rather than the access charge to help covering investment costs, i.e. public subsidies, in case of lack of demand for value-added services.
\[ E(q^*_F) = \begin{cases} 
\gamma \left( \frac{a-c+2\beta_2-\beta_1}{3} \right) + (1-\gamma) \left( \frac{a-c}{3} \right) & \text{if } 4\beta_1 - 5\beta_2 \leq a-c \\
\gamma(a-c+4\beta_2-3\beta_1) + (1-\gamma) \left( \frac{a-c}{3} \right) & \text{otherwise}
\end{cases} \]

From the above equations we can see that: when \(4\beta_1 - 5\beta_2 \leq a-c\), the expected equilibrium quantities are positive, given \(a-c > 0\) and Assumption 2.4; when \(4\beta_1 - 5\beta_2 > a-c\), on one side, the incumbent’s expected quantity is unambiguously positive - because \(a-c > 0\) and \(\beta_1 > \beta_2\) in this case -, and on the other side, the positive sign for the OLO’s quantity is guaranteed by Condition 2.4.\(^{11}\)

Notice that Condition 2.4 here implies that the regulator sets access conditions in such a way not to exclude the OLO from the market, when the OLO has a lower ability in offering value-added services with respect to the incumbent, although it is equally efficient on the cost side. This case appears to be more realistic and in line with the institutional framework in Europe.\(^{12}\)

Simple algebra identifies the range for \(\beta_2\) for which it is possible to have a positive regulated access price and the OLO active in the NGN market altogether. Such range of parameters is:

\[ \frac{3\beta_1}{4} - \frac{a-c}{6} \leq \beta_2 < \frac{4\beta_1}{5} - \frac{(a-c)}{5} \]

where the right hand side corresponds to the condition for an above cost access price, and the left hand side corresponds to the condition for non-exclusion of the OLO. This range of parameters exists only if \(\beta_1 > 2(a-c)/3\). For all \(\beta_1 \leq 2(a-c)/3\), the threshold value for \(\beta_2\) to have non-exclusion and positive access price is higher than the threshold necessary to have a positive regulated

\(^{11}\)Recall that Condition 2.4 ensures the ex post convenience for the OLO to use the NGN in any state of demand.

\(^{12}\)The European Commission (2002, page 117 - 119), indeed, has adopted the standard of Equally Efficient Operator (EEO) in the context of access regulation and price test. Besides that, demand factors are less observable and much more volatile, so we would not expect the regulator to base its decisions on access price on demand factors so heavily as to exclude an EEO from the market, most of all in a situation where uncertainty plays a central role.
access price in the first place, as shown in Figure 2.2.

Intuitively, as long as the OLO’s ability is higher than the incumbent’s one, the regulator favours the OLO’s participation into the market through a low access price, i.e. setting the access charge equal to the marginal cost. The regulator starts setting an above cost access charge when the incumbent’s ability in boosting the demand becomes considerably higher than the OLO’s one.\textsuperscript{13} In this case the OLO remains active in the market as long as its ability is above some minimum threshold, \(3\beta_1/4 - (a - c)/6 \leq \beta_2\).

**Stage 0: the incumbent chooses the investment timing**

The incumbent will have different objective functions depending on the parameters. When \(4\beta_1 - 5\beta_2 \leq a - c\), we have that \(r^F = 0\). Therefore the incumbent makes no profit in the upstream market and its objective function is:

\[
\max_{\Delta^F} E(\pi^F) = (1 - \Delta^F) \left(\frac{a - c}{3}\right)^2 + \Delta^F \left(\gamma \left(\frac{a - c + 2\beta_1 - \beta_2}{3}\right)^2 + (1 - \gamma) \left(\frac{a - c}{3}\right)^2\right) - (\Delta^F)^2 \phi/2
\]

When \(4\beta_1 - 5\beta_2 > a - c\), we have that \(r^F > 0\), then the incumbent’s objective function is:

\[
\max_{\Delta^F} E(\pi^F) = (1 - \Delta^F) \left(\frac{a - c}{3}\right)^2 + \Delta^F \left(\gamma (2(\beta_1 - \beta_2)^2 + (c - a + 4\beta_1 - 5\beta_2)(a - c + 4\beta_2 - 3\beta_1)) + (1 - \gamma) \left(\frac{a - c}{3}\right)^2\right) - (\Delta^F)^2 \phi/2
\]

The two first-order conditions with respect to investment timing \(\Delta^F\) give the following solution:

\[
\Delta^F = \begin{cases} 
\frac{(2(a-c)(2\beta_1-\beta_2)+(2\beta_1-\beta_2)^2)\gamma}{9\phi} & \text{if } 4\beta_1 - 5\beta_2 \leq a - c \\
\frac{(-72(\beta_1-\beta_2)^2+9(7\beta_1-9\beta_2)(a-c)+9\beta_2(7\beta_1-8\beta_2)-10(a-c)^2)\gamma}{9\phi} & \text{otherwise}
\end{cases}
\]

This is the incumbent optimal investment timing as long as the conditions \(((2(a - c)(2\beta_1 - \beta_2) + (2\beta_1 - \beta_2)^2)\gamma)/(9\phi) \leq 1\) and \(((−72(\beta_1 - \beta_2)^2 + 9(7\beta_1 - \beta_2)^2))\gamma)/(9\phi) \leq 1\).

\textsuperscript{13}This result is in line with Mizuno and Yoshino (2012), who also find that, when the degree of spillover is small, i.e. when the OLO has a lower ability to offer value-added services, the incumbent has the incentive to overinvest in order to obtain an above cost access charge from the regulator.
In line with Foros (2004), here we find that the optimal investment timing chosen by the incumbent is negatively correlated with the OLO’s ability to provide value-added services, i.e. $\frac{d}{d\beta_2} \Delta F^* < 0$. When the regulated access price is set equal to the marginal cost, the incumbent has no profit by leasing lines to the OLO in the upstream market. Therefore the incumbent’s investment is a pure spillover, increasing with the OLO’s ability to exploit the new technology. When the regulated access price is positive, the investment decreases with the OLO’s ability. So in both cases, the better is the OLO, the later the incumbent tends to invest.

When the probability of success increases, the incumbent’s incentive to invest in the NGN decreases less rapidly with the OLO’s ability, $\frac{d^2}{d\beta_2 d\gamma} \Delta F^* < 0$, but also the investment is made earlier $\frac{d}{d\gamma} \Delta F^* > 0$. This happens because, other things being equal, a higher probability of success gives the incumbent overall higher incentives to invest. Therefore, the effect for which an increase in the OLO’s ability determines a decrease in the incumbent’s investment incentive becomes less strong if the probability of success is higher.

The socially optimal investment timing

If we substitute all equilibrium solutions into the welfare function, the first-order condition with respect to $\Delta F^W$ gives the following result:

$$\Delta F^W = \begin{cases} \frac{8(a-c)(\beta_1 + \beta_2) + 11(\beta_2 - \beta_1)^2 + 8\beta_1\beta_2}{18\phi} \gamma & \text{if } 4\beta_1 - 5\beta_2 < a - c \\ \frac{(a-c)^2 + 9(2\beta_2 - \beta_1)^2 + 18\beta_1(\beta_1 - \beta_2) + 18\beta_2(a-c)}{18\phi} \gamma & \text{otherwise} \end{cases}$$

This is the socially optimal investment timing as long as the conditions $(8(a-c)(\beta_1 + \beta_2) + 11(\beta_2 - \beta_1)^2 + 8\beta_1\beta_2)\gamma/(18\phi) \leq 1$ and $((a-c)^2 + 9(2\beta_2 - \beta_1)^2 + 18\beta_1(\beta_1 - \beta_2) + 18\beta_2(a-c))\gamma/(18\phi) \leq 1$ are satisfied.

2.2.3 Partial Regulation

Stage 1: the incumbent chooses the access price to the NGN

The incumbent’s profit function after investment is:

$$E(\pi_1^P) = \gamma((q_1^{P^*})^2 + r^{P^*}q_2^{P^*}) + (1 - \gamma)(q_1^{P^*})^2 - \Delta\phi/2$$

Remind that $r^{P^*} = 0$, by Condition 2.3. We analyse the situation in which the incumbent makes a take-it-or-leave-it offer to the OLO, differently from Nitsche and Wietaus (2011) who model the partial regulation case as a Nash bargaining. Considering Condition 2.4, the incumbent’s profit maximisation gives three parameters range that determine different values for the access
price chosen by the firm, as shown in Figure 2.3:

\[ r^{P*} = \begin{cases} \frac{a-c}{2} + \frac{\beta_1+4\beta_2}{10} & \text{if } 2(\beta_2 - \beta_1)/5 \geq (a - c)/3 \text{ and } 6\beta_2 < 5\beta_1 \\ \frac{2\beta_2 - \beta_1}{2} & \text{if } 6\beta_2 \geq 5\beta_1 \end{cases} \]

Figure 2.3: Partial Regulation

When \( \beta_2 \) is higher than \( \beta_1 \) by a considerable extent, i.e. \( 2(\beta_2 - \beta_1)/5 \geq (a - c)/3 \), the OLO earns higher profits in the NGN market, paying the unregulated access charge, than in the outside option. Therefore, the incumbent charges the access price that maximises its profits and allows the greatest rent extraction from the OLO in the upstream market. The parameter threshold \( 2(\beta_2 - \beta_1)/5 \geq (a - c)/3 \) derives from Condition 2.4, once inserted the expression for the unregulated access price into the equilibrium quantities.

If \( 2(\beta_2 - \beta_1)/5 \geq (a - c)/3 \), the corresponding expected equilibrium quantities are the following:

\[ \begin{align*} E(q_1^{P*}) &= \gamma \left( \frac{a-c}{2} + \frac{2\beta_2 - \beta_1}{5} \right) + (1 - \gamma) \left( \frac{a-c}{3} \right) \\
E(q_2^{P*}) &= \gamma \left( \frac{2(\beta_2 - \beta_1)}{5} \right) + (1 - \gamma) \left( \frac{a-c}{3} \right) \end{align*} \]

For intermediate values of the quality parameters, the incumbent will lower the access price down to the point where Condition 2.4 is verified with equality, once considered the equilibrium quantities. When \( \beta_1 \) is not considerably higher than \( \beta_2 \) - as defined by the second parameter threshold \( 6\beta_2 \geq 5\beta_1 \) (see Appendix 2.A for the derivation of the parameters threshold) - the incumbent’s profit from charging the constrained access price to the NGN is higher than the profit from exclusion.

In this case, we have an intermediate parameters range such that \( 2(\beta_2 - \beta_1)/5 < (a - c)/3 \) and \( 6\beta_2 \geq 5\beta_1 \) (see Figure 2.3), that yields the following
expected equilibrium quantities:

\[
\begin{align*}
E(q^P_1^*) &= \gamma \left( \frac{a-c}{3} + \frac{\beta_1}{3} \right) + (1 - \gamma) \left( \frac{a-c}{3} \right) \\
E(q^P_2^*) &= \frac{a-c}{3}
\end{align*}
\]

Finally, when the incumbent is considerably better in offering value-added services, it prefers to exclude the OLO from the NGN market. Hence, for \(6\beta_2 < 5\beta_1\), we obtain:

\[
\begin{align*}
E(q^P_1^*) &= \gamma \left( \frac{a-c}{3} + \frac{2\beta_1}{3} \right) + (1 - \gamma) \left( \frac{a-c}{3} \right) \\
E(q^P_2^*) &= \frac{a-c}{3}
\end{align*}
\]

Notice that since the OLO’s outside option is using the copper network rather than leaving the market entirely, unlike in Foros (2004), the OLO gets better wholesale access conditions. In Foros (2004), the incumbent always charges the unconstrained access price, which excludes the entrant whenever the entrant’s ability to exploit the new network is not higher than the incumbent’s ability. In this setting, for the parameters range \(\beta_1 + 5(a-c)/6 > \beta_2 > \beta_1\), the OLO is better than the incumbent in offering value-added services, and the incumbent charges an access price which is lower than the unconstrained access price. Furthermore, for the parameters range \(\beta_1 > \beta_2 \geq 5\beta_1/6\), the incumbent is better than the OLO in offering value-added services, but it charges an access price that keeps the OLO active in the NGN market. Only for values of the parameters such that \(5\beta_1/6 > \beta_2\) there is exclusion of the OLO.

Proposition 2.1. Under the assumptions \(r^C = 0\) and \(2\beta_i \geq \beta_j\) (\(i, j = 1, 2\) with \(i \neq j\)), when the OLO has the outside option to use the regulated copper network rather than leaving the market entirely, there is a range of parameters for which, absent regulatory intervention, there is no exclusion in the provision of value-added services, even if the incumbent’s ability is higher than the OLO’s ability in offering such services.

Proof. See Appendix 2.A. \(\square\)

Stage 0: the incumbent chooses the investment timing \(\Delta\)

After inserting \(r^P*, q^P_1^*\) and \(q^P_2^*\) into the incumbent’s profit function, the first-order condition of the profit maximisation with respect to \(\Delta\) returns the
following investment timings:

\[
\Delta^*_P = \begin{cases} 
\frac{(25(a-c)^2 + 9(10\beta_1(a-c) + (3\beta_1 - 2\beta_2)^2 + 4\beta_1\beta_2))\gamma}{180\phi} & \text{if } 2(\beta_2 - \beta_1)/5 \geq (a-c)/3 \\
\frac{(3\beta_1^2 + 2(a-c)(\beta_1 + 2\beta_2))\gamma}{12\phi} & \text{if } 6\beta_2 \geq 5\beta_1 \\
\frac{(4\beta_1(a-c + \beta_1))\gamma}{9\phi} & \text{if } 6\beta_2 < 5\beta_1 
\end{cases}
\]

This is the optimal investment timing chosen by the incumbent as long as the conditions \(((25(a-c)^2 + 9(10\beta_1(a-c) + (3\beta_1 - 2\beta_2)^2 + 4\beta_1\beta_2))\gamma)/(180\phi) \leq 1, ((3\beta_1^2 + 2(a-c)(\beta_1 + 2\beta_2))\gamma)/(12\phi) \leq 1\) and \(((4\beta_1(a-c + \beta_1))\gamma)/(9\phi) \leq 1\) are satisfied.

We find that, when the OLO participates in the NGN market, the investment timing is positively correlated with its ability to provide value-added services, \(\frac{d}{d\gamma}\Delta^*_P > 0\). Since the incumbent seeks to capture some rent from the OLO, the higher the OLO’s ability is, the earlier the incumbent invests, hoping to earn from access rents in the upstream market, in case of successful investment. This effect is stronger, the higher the probability of success, \(\frac{d}{d\gamma}\Delta^*_P > 0\). Also, unsurprisingly, the investment is made earlier in time, the higher the probability of success, \(\frac{d}{d\gamma}\Delta^*_P > 0\).

**The socially optimal investment timing**

As a benchmark for comparison, we now evaluate the socially optimal investment timing. The social welfare function can be written as:

\[
E(W^P) = (1 - \Delta)WC + \Delta E(W^{NP})
\]

where \(E(W^{NP})\) is the after-investment expected welfare with partial regulation - the superscript \(N\) stands for NGN - and it is given by:

\[
E(W^{NP}) = \gamma \left( \frac{(q_1^{P*s} + q_2^{P*s})^2}{2} + \frac{(q_1^{P*s})^2}{2} + r^{P*s}q_2^{P*s} - (\Delta)\phi/2 + (q_2^{P*s})^2 \right) + (1 - \gamma) \left( \frac{(q_1^{Pf*s} + q_2^{Pf*s})^2}{2} + (q_1^{Pf*s})^2 - (\Delta)\phi/2 + (q_2^{Pf*s})^2 \right)
\]

The first term inside the brackets represents the consumer surplus, the last term is the OLO’s profit and the remaining ones are the profit earned by the incumbent. After inserting all equilibrium solutions into \(E(W^P)\), the first-order conditions with respect to \(\Delta^P\) yield the following results in the different cases:

\[
\Delta^{PW} = \begin{cases} 
\frac{(-5(a-c)^2 + 766(2\beta_2 - \beta_1)^2 + 55(3\beta_1 - 2\beta_2)^2 + 19(3\beta_1 - 2\beta_2)^2 (a-c))\gamma}{200\phi} & \text{if } 2(\beta_2 - \beta_1)/5 \geq (a-c)/3 \\
\frac{(9\beta_1^2 + 4(a-c)(\beta_1 + 2\beta_2))\gamma}{24\phi} & \text{if } 6\beta_2 \geq 5\beta_1 \\
\frac{(111(\beta_1 - \beta_2)^2 + 2(a-c)(\beta_1 + 2\beta_2) + 8(\beta_1\beta_2))\gamma}{18\phi} & \text{if } 6\beta_2 < 5\beta_1 
\end{cases}
\]
The superscript \( W \) stands for the welfare maximising result. This is the socially optimal investment timings as long as the conditions \((-5(a-c)^2/72\phi) + (76(\beta_2 - \beta_1)^2 + \beta_1(55\beta_1 + 20\beta_2) + (130\beta_1 + 20\beta_2)(a-c)\gamma/(200\phi)) \leq 1, (9\beta_1^2 + 4(a-c)(3\beta_1 + 2\beta_2)\gamma)/(48\phi) \leq 1 \) and \(((11(\beta_1 - \beta_2)^2 + 8(a-c)(\beta_1 + \beta_2) + 8\beta_1\beta_2)\gamma)/(18\phi) \leq 1 \) are satisfied.

### 2.2.4 Extension: Compulsory switch to NGN

In this extension we show what happens to the incumbent’s access price decisions when there is compulsory switch to the NGN.\(^{14}\) In this case, the OLO’s outside option would be exiting the market, as in Foros (2004). When the OLO’s alternative is leaving the market entirely, the only circumstance under which the OLO makes positive profits in the NGN is when it has more ability to exploit the new network than the incumbent. When \( \beta_2 < \beta_1 \), indeed, the incumbent is indifferent between charging an access price that extracts the OLO’s profits entirely, or one that fully excludes the OLO from the NGN market.

**Stage 2**

Equilibrium quantities in stage 2 are unchanged compared to our basic model.

The ex post participation conditions are different, since the copper network option is not available anymore once the NGN investment is deployed. The outside option scenario consists in the OLO exiting the market and the incumbent being monopolist:

\[
\pi_{1}^{os} = \left(\frac{a-c + \beta_1}{2}\right)^2, \pi_{2}^{os} = 0
\]

\[
\pi_{1}^{of} = \left(\frac{a-c}{2}\right)^2, \pi_{2}^{of} = 0
\]

The ex post OLO’s participation conditions are the following:

- in case of success
  \[
  \left(\frac{a-c-2r_{ls} + 2\beta_2 - \beta_1}{3}\right)^2 \geq 0
  \]

- in case of failure
  \[
  \left(\frac{a-c-2r_{lf}}{3}\right)^2 \geq 0
  \]

\(^{14}\)At present, mandatory switch of the legacy network is not included in the EU regulatory framework.
The above conditions require that:

\[ r^{ls} \leq \frac{a-c+2\beta_2-\beta_1}{2} \]
\[ r^{lf} \leq \frac{a-c}{2} \]

with \( l = P, F, RS \).

**Stage 1: the incumbent chooses the access price to the NGN**

The incumbent’s profit function after investment is unchanged:

\[ E(\pi_1^P) = \gamma ((q_1^{P*})^2 + r^{P*}q_2^{P*}) + (1 - \gamma) (q_1^{P*})^2 - \Delta \phi/2 \]

The expected access price chosen by the firm is the following:

\[ r^{P*} = \begin{cases} 
\frac{a-c+\beta_1+4\beta_2}{10} & \text{in case of success} \\
\frac{a-c}{2} & \text{in case of failure}
\end{cases} \]

The corresponding expected equilibrium quantities are the following:

\[ \begin{cases} 
E(q_1^{P*}) = \gamma \left( \frac{a-c}{2} + \frac{7\beta_1-2\beta_2}{10} \right) + (1 - \gamma) \left( \frac{a-c}{2} \right) \\
E(q_2^{P*}) = \gamma \left( \frac{2\beta_2-\beta_1}{5} \right)
\end{cases} \]

As we can see, the incumbent always sells positive quantities, but the OLO has non-negative quantities only if \( \beta_2 > \beta_1 \), i.e. with this access price level, whenever the OLO is not at least as good as the incumbent in offering value-added services, it will be excluded from the market. Alternatively, the incumbent can charge the constrained access price that verifies the OLO’s ex post access condition with equality.

In the following we prove that, when \( \beta_2 \leq \beta_1 \), the incumbent is indifferent between charging the unconstrained access price that excludes the OLO and charging the constrained access price that verifies the OLO’s ex post participation constraints with equality, \( r^{Pconst*} \), which is:

\[ r^{Pconst*} = \begin{cases} 
\frac{a-c+2\beta_2-\beta_1}{2} & \text{in case of success} \\
\frac{a-c}{2} & \text{in case of failure}
\end{cases} \]

The constrained access price in the equation above yields the following expected equilibrium quantities:

\[ \begin{cases} 
E(q_1^{P*}) = \gamma \left( \frac{a-c+\beta_1}{2} \right) + (1 - \gamma) \left( \frac{a-c}{2} \right) \\
E(q_2^{P*}) = 0
\end{cases} \]
Therefore, the incumbent’s profits from exclusion, \( \pi^o_1 = \gamma \pi^{os}_1 + (1 - \gamma) \pi^{off}_1 \), or from market sharing with the constrained access price, \( \pi^{Ps}_1[\frac{r}{P_{const}}] \), are the same:

\[
\pi^o_1 = \gamma \left( \frac{a - c + \beta_1}{2} \right)^2 + (1 - \gamma) \left( \frac{a - c}{2} \right)^2
\]

\[
\pi^{Ps}_1[\frac{r}{P_{const}}] = \gamma \left( \frac{a - c + \beta_1}{2} \right)^2 + (1 - \gamma) \left( \frac{a - c}{2} \right)^2
\]

When the OLO’s outside option is exiting the market entirely, if we assume that when indifferent the incumbent favors market sharing, there is no case for exclusion with partial regulation.

The access conditions though are less favorable to the OLO. Whenever the OLO is not at least as good as the incumbent in offering value-added services, its profits are driven down to zero. In our basic model instead, we find that there is a case in which the OLO is worse than the incumbent in offering value-added services but it earns positive profits and remains active in the market.

2.2.5 Risk Sharing

We model the risk sharing agreement as an exogenous alternative, to highlight its potential improvements over social welfare outcomes. More specifically, following Nitsche and Wiethaus (2011), the risk sharing option is treated in a reduced form in which parties share the fixed cost of investment through some agreement and then they can use the NGN network without further side payments. In this respect, risk sharing may be thought as a compulsory regime imposed on firms by the regulator.\(^{15}\)

In this setting we do not have the choice of access price, because firms first compete on services using the copper network and then use the commonly built NGN, without further side payments for the network usage. Therefore we can directly analyse the choice of investment timing.

**Stage 0: Joint choice of investment timing**

The expected equilibrium quantities in the last stage of the risk sharing

---

\(^{15}\)We do not address the specific nature of the risk sharing contracts in this paper. On this point, Inderst and Peitz (2012b) analyse cost-sharing agreements between an incumbent firm and an entrant, in the form of long term contracts concluded before the investment is made, as opposed to contracting taking place after the network has been constructed. The authors show that the former type of agreement reduces the duplication of investment and may lead to more areas being covered. Coordination at the investment level may come at a price, though, which is reduced competition in the areas thus covered.
game write as below:

\[
\begin{align*}
E(q_{RS}^1) &= \gamma \left( \frac{a-c+2\beta_1-\beta_2}{3} \right) + (1-\gamma) \left( \frac{a-c}{3} \right) \\
E(q_{RS}^2) &= \gamma \left( \frac{a-c+2\beta_2-\beta_1}{3} \right) + (1-\gamma) \left( \frac{a-c}{3} \right)
\end{align*}
\]

Assumption 2.4 ensures that both firms are active in the market, in every state of demand.

The two firms choose the investment timing by maximising the sum of their expected profits, \(E(\pi_{RS}^{12})\), considering the equilibrium quantities in the retail market:

\[
\max_{\Delta RS} E(\pi_{RS}^{12}) = (1 - \Delta RS) \frac{2(a-c)^2}{9} + \\
\Delta RS \left( \gamma \left( \frac{(a-c+2\beta_1-\beta_2)^2}{9} + \frac{(a-c+2\beta_2-\beta_1)^2}{9} \right) + \gamma \right) + \\
(1-\gamma) \frac{2(a-c)^2}{9} - (\Delta RS)^2 \phi / 2
\]

Their choice yields the following timing for the investment in the NGN:

\[
\Delta RS^* = \frac{(2(a-c)(\beta_1 + \beta_2) + 5(\beta_1 - \beta_2)^2 + 2\beta_1 \beta_2)\gamma}{9\phi}
\]

\(\Delta RS^*\) is the optimal timing of investment when incumbent and OLO enter in a cooperation agreement for the construction of the NGN infrastructure only if \((2(a-c)(\beta_1 + \beta_2) + 5(\beta_1 - \beta_2)^2 + 2\beta_1 \beta_2)\gamma)/(9\phi) \leq 1\). The second-order condition is always satisfied. Notice that the optimal \(\Delta RS^*\) would be zero if there were no expected demand increase following the investment, i.e. \(\beta_1 = \beta_2 = 0\). Of course, the two firms would have no interest in investing in the NGN technology if they believed there would be no market for their value-added services.

Furthermore, it is interesting to analyse how such choice changes with the difference in the ability to offer value-added services and therefore with the returns from the investment. Comparative statics shows that the sign of \(\frac{d}{d\beta_2} \Delta RS^*\) depends on the term \(5\beta_2 - 4\beta_j + a - c\), with \(i, j = 1, 2\) and \(i \neq j\). Keeping \(\beta_1\) fixed, an increase in the value of \(\beta_2\) unambiguously yields to anticipating the joint construction of the NGN, i.e. \(\frac{d}{d\beta_2} \Delta RS^* > 0\), when \(5\beta_2 - 4\beta_1 + a - c \geq 0\), therefore, only when the OLO is better than the incumbent, or when the incumbent is better than the OLO but not too much. When \(5\beta_2 - 4\beta_1 + a - c < 0\), the incumbent is considerably better than the OLO in offering value-added services and an increase in the ability of the OLO delays the construction of the NGN, i.e. \(\frac{d}{d\beta_2} \Delta RS^* < 0\). This effect reflects the fact that, with risk sharing, the two firms internalise the profit externalities generated by Cournot competition.
Notice, indeed, that we encountered the same conditions for the solution to the first-order condition in case of full regulation: \( r^{F*} = 0 \) if \( 5\beta_2 - 4\beta_1 + a - c \geq 0 \) and \( r^{F*} > 0 \) if \( 5\beta_2 - 4\beta_1 + a - c < 0 \).

**The socially optimal investment timing**

As a benchmark for comparison, we compute the socially optimal investment timing in case of risk sharing, which is:

\[
\Delta_{RSW} = \frac{(8(a-c)(\beta_1 + \beta_2) + 11(\beta_1 - \beta_2)^2 + 8\beta_1\beta_2)\gamma}{18\phi}
\]

The equation above represents the socially optimal investment timing in case of risk sharing as long as \((8(a-c)(\beta_1 + \beta_2) + 11(\beta_1 - \beta_2)^2 + 8\beta_1\beta_2)\gamma) / 18\phi \leq 1\).

### 2.2.6 Comparison of results under partial regulation, full regulation and risk sharing

We can derive the first insight from this model by comparing the results obtained in case of partial access regulation, full access regulation and risk sharing.

**Proposition 2.2.** For a given timing of investment \( \Delta \) and under the assumptions \( r^F \geq 0 \) and \( 2\beta_i \geq \beta_j \) \((i, j = 1, 2 \text{ with } i \neq j)\), expected industry output \( E(Q(\Delta)) \) satisfies

\[
E(Q^{RS}(\Delta)) > E(Q^P(\Delta)) \geq E(Q^F(\Delta))
\]

**Proof.** See Appendix 2.A.

In line with Nitsche and Wiethaus (2011), risk sharing is expected to induce more competition than partial regulation and full regulation regimes. The first inequality \( E(Q^{RS}(\Delta)) > E(Q^P(\Delta)) \) arises because risk sharing involves no wholesale transfers and a more symmetric market structure,\(^\text{16}\) whereas partial regulation implies transfer from the OLO to the incumbent and an asymmetric market structure, which reflects the lower level of competition. The second

\(^{16}\text{The possible difference in market shares reflects only the differences in abilities, not differences in market power. If the two firms are equal in abilities, market structure is symmetric under risk sharing.}\)
inequality \( E(Q^{RS}(\Delta)) \geq E(Q^F(\Delta)) \) arises because, when the regulated access price is constrained to zero by Assumption 3.1, risk sharing and full regulation yield the same outcome in terms of expected total quantities, but when the regulated access price is positive, full regulation involves a positive transfer which is higher than marginal cost of production, so the overall market efficiency is higher under risk sharing.

The equilibrium results in terms of NGN access conditions and, consequentially, investment incentives, change depending on the relative and absolute value of firms’ abilities in offering value-added services. In Table 2.1, we combine the various modes of regulation’s equilibrium outcomes, identifying five different relevant parameters ranges. For ease of exposition, we name them as following: \( P1F1RS, P2F1RS, P3F1RS, P3F2RS, P3F3RS \).

Case \( P1F1RS \) describes the situation in which the OLO has considerably more ability than the incumbent in offering value-added services through the NGN. In this case, when the access price is not regulated, the incumbent chooses the monopoly price, whereas the regulator would choose a negative access price that we constrained to zero by Assumption 3.1. In the second case, \( P2F1RS \), the values of the two firms’ abilities are close to each other, either favoring the incumbent or the OLO. Here, with partial regulation, the incumbent chooses to charge a constrained access price that makes it indifferent for the OLO to use the NGN or switch back to the copper network, while the full regulation outcome is unchanged compared to the previous situation. As the OLO’s ability decreases with respect to the incumbent’s one, the incumbent finds it less convenient to share the NGN market with the OLO, up to a point where it prefers to provide the value-added services alone. Therefore in the range of values \( P3F1RS \), we obtain exclusion with partial regulation, while the access price is zero with full regulation. When the incumbent becomes considerably better than the OLO in boosting the demand, the regulator favors its activity by imposing a positive regulated access price, but only insofar as that does not exclude the OLO from the market - case \( P3F2RS \). A positive regulated access price together with non-exclusion is not possible if the difference between the two firms’ abilities is important but their absolute values are low. In that case, the OLO would prefer to use the regulated copper network if asked to pay for the NGN, as in case \( P3F3RS \) where we have double exclusion, with full regulation and with partial regulation. We do not look into this case, as explained in section 2.2.2.

**Proposition 2.3.** Under the assumptions \( r^F \geq 0, 2\beta_i \geq \beta_j \) \((i, j = 1, 2 \text{ with } i \neq j)\), and given the OLO’s participation constraints (2.2) and (2.4), the following results hold:

1. Both firms are active in the market no matter what is the regulatory regime, for \( \beta_2 \geq 5\beta_1/6 \)
2. The OLO is excluded from the NGN market with partial regulation, for \( \beta_2 < 5\beta_1/6 \)
3. The investment is undertaken later with full regulation and risk sharing with respect to partial regulation: \( \Delta^{F*} < \Delta^{P*}; \Delta^{RS*} < \Delta^{P*} \)
Table 2.1: Relevant Parameters Thresholds

<table>
<thead>
<tr>
<th>Parameters Range</th>
<th>Partial Regulation</th>
<th>Full Regulation</th>
<th>Risk Sharing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g &gt; \beta_1 + \frac{a-c}{6}$</td>
<td>$P^1$: $E(r^*)$ unconstrained, OLO in the NGN market</td>
<td>$F^1$: $E(r^*) = 0$</td>
<td>$RS$: no upstream transfers</td>
</tr>
<tr>
<td>$\beta_1 + \frac{a-c}{6} &gt; \beta_2 \geq \beta_1 + \frac{5(a-c)}{6}$</td>
<td>$P^2$: $E(r^*)$ constrained, OLO in the NGN market</td>
<td>$F^1$: $E(r^*) = 0$</td>
<td>$RS$: no upstream transfers</td>
</tr>
<tr>
<td>$5\beta_1 &gt; \beta_2 \geq \frac{5\beta_1}{4} - \frac{a-c}{5}$</td>
<td>$P^3$: $E(r^*)$ unconstrained, OLO's EXCLUSION</td>
<td>$F^1$: $E(r^*) = 0$</td>
<td>$RS$: no upstream transfers</td>
</tr>
<tr>
<td>$\frac{3\beta_1}{2} - \frac{a-c}{3}$</td>
<td>$P^4$: $E(r^*)$ unconstrained, OLO's EXCLUSION</td>
<td>$F^1$: $E(r^*) = 0$, OLO in the NGN market</td>
<td>$RS$: no upstream transfers</td>
</tr>
<tr>
<td>$\frac{3\beta_1}{2} - \frac{a-c}{3} &gt; \beta_2 &gt; 0$</td>
<td>$P^5$: $E(r^*)$ unconstrained, OLO's EXCLUSION</td>
<td>$F^1$: $E(r^*) = 0$, OLO's EXCLUSION</td>
<td>$RS$: no upstream transfers</td>
</tr>
</tbody>
</table>

Figure 2.4: Relevant parameter thresholds
4. The investment is undertaken later with full regulation with respect to risk sharing when the regulated access price is zero, while the relationship is ambiguous when the regulated access price is positive: for \( \beta_1 \leq 2(a - c)/3 \), \( \Delta F^* < \Delta RS^* \); \( \Delta F^* \leq \frac{2}{3} \Delta RS^* \), for \( \beta_1 > 2(a - c)/3 \).

5. The OLO’s ability to provide value-added services through the NGN affects the investment timing. The effect is positive with partial regulation, \( \frac{d}{d \beta_2} \Delta P^* > 0 \); negative with full regulation, \( \frac{d}{d \beta_2} \Delta F^* < 0 \); ambiguous with risk sharing, where the impact of a firm’s ability to offer value-added services on the investment timing changes from positive to negative as \( \beta_2 \)’s absolute value decreases with respect to \( \beta_1 \), or vice versa.

6. The investment is undertaken later with respect to the social optimum, under all regulatory regimes: \( \Delta P^* < \Delta PW^* \); \( \Delta F^* < \Delta FW^* \); and \( \Delta RS^* < \Delta RSW^* \).

Proof. See Appendix 2.A.

The OLO always benefits from a spillover effect from the construction of the NGN done by the incumbent. Nevertheless, the incumbent can potentially capture some rent by leasing its infrastructure to the rival. Under full access price regulation though, when the OLO has more ability than the incumbent, the rent is set just equal to the marginal cost by the regulator, so the incumbent earns nothing from the upstream market. In this case, its investment incentive is dampened, since it cannot extract any benefit from the OLO’s value-added services. Therefore, it chooses to invest later with respect to the case without regulation. This confirms the finding in the literature that access price regulation plays a disincentive role in the incumbent’s investment decision (Kotakorpi (2006)). It is worth pointing out that, although in this model firms earn positive profits in the full regulation case thanks to the Cournot competition assumption, a similar result is found in a setting with Hotelling competition by Kotakorpi (2006). Less uncertainty mitigates such effect: when the investment success becomes more likely, the speed at which the incumbent delays its investment plans when \( \beta_2 \) is higher decreases, \( \frac{d^2}{d \beta_2^2} \Delta F^* < 0 \).

When \( \beta_1 > 2(a - c)/3 \), there is a range of parameters, \( 3\beta_1/4 - (a - c)/6 \leq \beta_2 < 4\beta_1/5 - (a - c)/5 \), for which the regulated access price is positive, the OLO is active in the NGN market with full regulation, but it is excluded with partial regulation. In this case, partial regulation gives the highest investment incentive, but the relationship between full regulation and risk sharing in terms of investment timing is ambiguous.

2.2.7 Welfare analysis

The previous analysis revealed that risk sharing induces the highest expected level of competition downstream for a given investment timing, in line with Nitsche and Wiethaus (2011), while partial regulation gives the strongest investment incentive. In this section, we provide a comprehensive welfare ranking of the different modes of regulation, broken down according to the range of parameter values shown in Figure 2.4. In the Appendix A.1.4 we report a detailed overview of the results. From these results, we derive the following statement.

**Proposition 2.4.** Under the assumptions \( r^F \geq 0 \), \( 2\beta_i \geq \beta_j \) (\( i, j = 1, 2 \) with \( i \neq j \)), and given the OLO’s participation constraints (2.2) and (2.4), the following results hold:

1. Expected consumer welfare is higher under risk sharing compared to partial regulation;
2. When the OLO is better than the incumbent in offering value-added services, expected total welfare is higher under risk sharing compared to partial regulation;

3. When the OLO is better than the incumbent in offering value-added services or when the incumbent is better than the OLO by a great extent, expected consumer welfare and expected total welfare are higher under partial regulation compared to full regulation. Otherwise, the difference in total welfare and consumer welfare between partial and full regulation remains ambiguous.

4. When the access price to the NGN is regulated at marginal cost, expected consumer welfare and expected total welfare are higher under risk sharing compared to full regulation;

Proof. See Appendix 2.A.

Once taken into account the equilibrium choice of investment timing, we find that risk sharing yields a higher expected consumer surplus than full regulation. When the regulated access price is zero, risk sharing also unambiguously yields a higher overall welfare than full regulation. However, when comparing partial regulation and risk sharing, investment incentives and intensity of competition move in opposite directions, therefore the results in terms of expected consumer welfare and expected total welfare change depending on the parameter values.

In particular, when the OLO is better in offering value-added services, the incumbent charges an access price that ensures the OLO’s participation to the NGN with partial regulation, while the access price is set to marginal cost with full regulation, i.e. cases \( P_1F1RS \) and \( P_2F1RS \) with \( \beta_2 \geq \beta_1 \). Under these circumstances, risk sharing is unambiguously dominant, both from a total welfare and a consumer welfare viewpoint.

Even though risk sharing investment incentives are lower compared to partial regulation, the higher competitive intensity more than compensates for the delay in building the NGN.

When the incumbent is better in offering value-added services, the welfare analysis becomes less clear. In the range of parameters for which the incumbent charges a constrained access fee and both firms are active in the NGN market, i.e. \( P2F2RS \) with \( \beta_2 < \beta_1 \), we find that full regulation still yields the least desirable outcome, but the relationship between partial regulation and risk sharing is ambiguous both from a consumer welfare and a total welfare viewpoint. The trade-off between stronger investment incentives under partial regulation and higher competitive intensity under risk sharing is less stark when the incumbent charges the lower constrained access fee. Therefore, depending on the parameters, total welfare can be higher or lower under risk sharing or partial regulation.

Finally, we analyse two cases in which the incumbent finds it more convenient to exclude the OLO from the NGN market. In this case, the OLO offers broadband services through the copper network, earning positive profits thanks to the regulated access price. Under this circumstance, when the incumbent’s ability in offering value-added services is not too high, i.e. \( \beta_1 \leq 2(a-c)/3 \), there is exclusion with partial regulation and a marginal cost access pricing with full regulation, i.e. \( P3F1RS \). Risk sharing is still unambiguously better than full regulation, both from a total welfare and a consumer welfare perspective. The relationship between partial regulation and risk sharing in terms of total welfare outcome is ambiguous.

When the incumbent’s ability in offering value-added services is high enough, i.e. \( \beta_1 > 2(a-c)/3 \), there is exclusion with partial regulation, and, for a certain range
of parameters, the regulator sets an above cost access price to the NGN and the OLO remains active in the NGN market, i.e. $P^F2RS$, with full regulation. In this case, partial regulation investment incentives are so elevated that total welfare turns out to be the highest compared to risk sharing and full regulation. The relationship between risk sharing and full regulation in terms of total welfare is ambiguous: investment’s incentives can be higher or lower depending on the parameters, but consumer welfare is always higher with risk sharing.

2.3 Conclusion

In this paper we model the competition between a vertically-integrated incumbent firm and a facilities-free OLO in the broadband market, where the former has the option to invest in building a NGN that allows firms to drastically increase the quality and variety of their services. Market success of the NGN in terms of demand increase is uncertain. Differently from other studies that assume demand uncertainty, the investment choice is analysed in a dynamic setting with differentiated products. The analysis is conducted under three different possible modes of regulation: full regulation (the NGN is regulated), partial regulation (the NGN is unregulated) and risk sharing (there are no side payments between firms for the use of the NGN).

Our analysis reveals that the investment is always undertaken later than in the social optimum in all modes of regulation. The investment choice is affected by the OLO’s ability to offer value-added services. Such effect is positive with partial regulation and negative with full regulation, while with risk sharing the effect changes from positive for high values of the OLO’s ability, $\beta_2$, to negative as the incumbent’s ability, $\beta_1$ gets considerably bigger than $\beta_2$, and vice versa. Partial regulation always yields the earliest investment compared to the other regulatory regimes, while risk sharing ensures the highest level of competitive intensity.

Welfare outcomes reveal that risk sharing is the dominant regime in a consumer surplus perspective. Expected consumer surplus is always higher under risk sharing than under partial regulation, but also under full regulation for a large set of parameters. In particular, when both firms are active, full regulation’s consumer surplus outcome is the least preferable; only when the incumbent’s ability is so high that regulated access price to the NGN is above marginal cost, the comparison of outcomes in terms of consumer surplus between full regulation and risk sharing becomes ambiguous.

Furthermore, when the OLO is better in offering value-added services, risk sharing is the dominant regime also from a total welfare perspective. When the incumbent is better, instead, welfare comparisons between the three regulatory regimes become less clear.

It is worth pointing out that these results are valid for the reduced form of risk sharing that we have considered in this paper. Such form of risk sharing implies a long term contract with no side payments for the use of the NGN, thereby excluding sources of inefficiency from the market and increasing the level of competitiveness downstream. More complicated contractual forms of risk sharing might arise in reality, which could well make the welfare comparison with full regulation less favorable to risk sharing. In terms of policy recommendations, we can state that, if risk sharing works smoothly, as in the model, then it allows for welfare improvements compared to full regulation. While, indeed, looking deeper into how risk sharing works is worth additional future
Our analysis sheds some conceptual light on the debate about what is the socially preferable access regulation regime to prompt telecommunications network development. The difference in firms’ ability to provide value-added services is important in the context. It exerts influence on the investment choice and on the access pricing decisions, which in turn affect market competition and social welfare. We find that demand uncertainty requires a careful formulation of access regulation rules. A robust set of rules should take into account the potential for an investment failure and provide reasonable access conditions for the firms involved in all possible cases. Also, uncertainty plays the role of delaying the investment decision in all regimes. According to our analysis, risk sharing can be particularly beneficial for consumers and give fairly high investment incentives at the same time. At this stage, it would also be interesting to go further in the research to study how risk sharing agreement can be robust to the inclusion of late entrants, to avoid that the construction of the NGN could possibly become a new source of market power and thereof be unable to deploy all of its benefits. It would also be interesting to make the choice to engage in a risk sharing agreement endogenous. We leave these questions for future research.
Appendix 2.A  Proofs

2.A.1  Proof of Proposition 2.1

When \(2(\beta_2 - \beta_1)/5 < (a - c)/3\), partial regulation unconstrained access price gives the OLO less profits than the outside option. The access price that verifies the OLO’s participation constraint 2.4 with equality is:

\[
a - c + r^{Ps} + 2\beta_2 - \beta_1 = \frac{a - c}{3} \tag{2.7a}
\]

\[
r^{Ps} = \frac{2\beta_2 - \beta_1}{2}
\]

The incumbent will prefer to charge the access price corresponding to the equation above, rather than to charge the unconstrained access price and exclude the OLO, as long as the outside option profits from being the only provider of the value-added services through the NGN are not higher than the market sharing profits:

\[
\pi^{Ps}_{1[a, Ps* \geq 2(\beta_2 - \beta_1) \leq \frac{a - c}{3}] \geq \pi^o}
\]

\[
\left(\frac{a-c}{3} + \frac{\beta_1}{2}\right)^2 + \frac{(2\beta_2 - \beta_1)(a-c)}{3} \geq \left(\frac{a-c}{3} + \frac{2\beta_1}{3}\right)^2
\]

The above inequality is unambiguously satisfied only for values of \(\beta\)'s such that the incumbent’s advantage in ability to offer value-added services is not too large:

\[6\beta_2 \geq 5\beta_1\]

2.A.2  Proof of Proposition 2.2

Total expected quantities for a given investment timing, under the different modes of regulation are the following:

- **Partial regulation**

  \[
  \begin{align*}
  E(Q^P) &= (1 - \Delta^P)\frac{a-c}{3} + \Delta^P \left(\frac{2(a-c)}{3} + \left(\frac{3\beta_1 + 2\beta_2}{10} - \frac{a-c}{6}\right)\gamma\right) \quad \text{if } \frac{4\beta_1 - 5\beta_2}{a-c} \geq \frac{a-c}{3} \\
  E(Q^P) &= (1 - \Delta^P)\frac{a-c}{3} + \Delta^P \left(\frac{2(a-c)}{3} + \frac{\beta_1\gamma}{2}\right) \quad \text{if } 6\beta_2 \geq 5\beta_1 \\
  E(Q^P) &= (1 - \Delta^P)\frac{a-c}{3} + \Delta^P \left(\frac{2(a-c)}{3} + \frac{\beta_2\gamma}{2}\right) \quad \text{if } 6\beta_2 < 5\beta_1
  \end{align*}
  \]

- **Full regulation**

  \[
  \begin{align*}
  E(Q^F) &= (1 - \Delta^F)\frac{a-c}{3} + \Delta^F \left(\frac{2(a-c)}{3} + \frac{\beta_1\gamma}{3}\right) \quad \text{if } 4\beta_1 - 5\beta_2 \leq a-c \\
  E(Q^F) &= (1 - \Delta^F)\frac{a-c}{3} + \Delta^F \left(\left(2\beta_2 - \beta_1\right)\gamma + \frac{(a-c)(2+\gamma)}{4}\right) \quad \text{otherwise}
  \end{align*}
  \]

- **Risk sharing**

  \[
  E(Q^{RS}) = (1 - \Delta^{RS})\frac{a-c}{3} + \Delta^{RS} \left(\frac{2(a-c)}{3} + \left(\frac{\beta_1 + \beta_2}{2}\right)\gamma\right)
  \]

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We now compare partial regulation and full regulation with risk sharing, considering the specific conditions under each relevant parameter threshold, as defined in Table 2.1, and Assumption 2.4:

• if $P1RS$:

$$
\frac{E(Q^{RS})}{E(Q^P)} = \frac{10(2(a - c) + (\beta_1 + \beta_2)\gamma)}{20(a - c) + (3\beta_1 + 2\beta_2) - 5(a-c)\gamma} \\
10(2(a - c) + (\beta_1 + \beta_2)\gamma) - (20(a - c) + (3(\beta_1 + 2\beta_2) - 5(a-c))\gamma) = (\beta_2 - \frac{\beta_1}{2})\gamma > 0
$$

• if $P2RS$:

$$
\frac{E(Q^{RS})}{E(Q^P)} = \frac{2(2(a - c) + (\beta_1 + \beta_2)\gamma)}{4(a - c) + 3\beta_1\gamma} \\
2(2(a - c) + (\beta_1 + \beta_2)\gamma) - (4(a - c) + 3\beta_1\gamma) = (2\beta_2 - \beta_1)\gamma > 0
$$

• if $P3RS$:

$$
\frac{E(Q^{RS})}{E(Q^P)} = \frac{2(a - c) + (\beta_1 + \beta_2)\gamma}{2(a - c) + \beta_1\gamma} \\
2(a - c) + (\beta_1 + \beta_2)\gamma - (2(a - c) + \beta_1\gamma) = \beta_2\gamma > 0
$$

• if $F1RS$:

$$
\frac{E(Q^{RS})}{E(Q^P)} = 1
$$

• if $F2RS$:

$$
\frac{E(Q^{RS})}{E(Q^P)} = \frac{2(a - c) + (\beta_1 + \beta_2)\gamma}{2(a - c) + (3(\beta_2 - \beta_1) + (a-c))\gamma} \\
2(a - c) + (\beta_1 + \beta_2)\gamma - (2(a - c) + (3(\beta_2 - \beta_1) + (a-c))\gamma) = (4\beta_1 - 5\beta_2 - (a-c))\gamma > 0
$$

Therefore, $E(Q^{RS}) > E(Q^P)$; and $E(Q^{RS}) \geq E(Q^P)$.

2.A.3 Proof of Proposition 2.3

(1) and (2) Proof of these statements derives directly from Proposition 2.1.

(3) Investment timing: partial regulation vs full regulation and risk sharing

In order to compare investment timings we do the following computations, considering each time the specific conditions under each relevant parameter threshold, as defined in Table 2.1, and Assumption 2.4:

• if $P1F1$

$$
\frac{\Delta^P}{\Delta^F} = \frac{25(a - c)^2 + 90\beta_1(a - c) + 81\beta_1^2 - 72\beta_1\beta_2 + 36\beta_2^2}{20(2\beta_1 - \beta_2)(2(a - c)(2\beta_1 - \beta_2))} \\
25(a - c)^2 + 90\beta_1(a - c) + 81\beta_1^2 - 72\beta_1\beta_2 + 36\beta_2^2 - (20(2\beta_1 - \beta_2)(2(a - c)(2\beta_1 - \beta_2))) = \\
(5(a-c) + \beta_1 + 4\beta_2)^2 > 0
$$
In order to compare investment timings we do the following computations, considering Investment timing: risk sharing vs full regulation

\[
\Delta P = \frac{3(3\beta_1^2 + 2(\beta_1 + 2\beta_2)(a - c))}{4(2\beta_1 - \beta_2)(2(a - c) + 2\beta_1 - \beta_2)}
\]

\[
3(3\beta_1^2 + 2(\beta_1 + 2\beta_2)(a - c)) - (4(2\beta_1 - \beta_2)(2(a - c) + 2\beta_1 - \beta_2)) = 4(2\beta_2 - \beta_1)(10(a - c) + 7\beta_1 - 2\beta_2) > 0
\]

\[
\Delta P = \frac{4(a - c + \beta_1^2)}{(2\beta_1 - \beta_2)(2(a - c) + 2\beta_1 - \beta_2)}
\]

\[
(4(a - c + \beta_1^2) - (2\beta_1 - \beta_2)(2(a - c) + 2\beta_1 - \beta_2) = \beta_2(a - c + 4\beta_1 - \beta_2) > 0
\]

\[
\Delta P = \frac{-4\beta_1(a - c + \beta_1^2)}{-72(\beta_1 - \beta_2)^2 + 9(7\beta_1 - 8\beta_2)\beta_2 + 9(7\beta_1 - 9\beta_2)(a - c) - 10(a - c)^2}
\]

\[
4\beta_1(a - c + \beta_1^2) - (-72(\beta_1 - \beta_2)^2 + 9(7\beta_1 - 8\beta_2)\beta_2 + 9(7\beta_1 - 9\beta_2)(a - c) - 10(a - c)^2) = 10(a - c)^2 + 76(\beta_1 - \beta_2)^2 + \beta_2(-55\beta_1 + 68\beta_2) + (a - c)(-59\beta_1 + 81\beta_2) > 0
\]

\[
\Delta P = \frac{25(a - c)^2 + 90\beta_1(a - c) + 81\beta_1^2 - 72\beta_1\beta_2 + 36\beta_2^2}{20(5(\beta_1 - \beta_2)^2 + 2(a - c)(\beta_1 + \beta_2) + 2\beta_1\beta_2)}
\]

\[
25(a - c)^2 + 90\beta_1(a - c) + 81\beta_1^2 - 72\beta_1\beta_2 + 36\beta_2^2 - (20(5(\beta_1 - \beta_2)^2 + 2(a - c)(\beta_1 + \beta_2) + 2\beta_1\beta_2)) = 25(a - c)^2 + 10(5\beta_1 - 4\beta_2)(a - c) - 4(5\beta_1 - 4\beta_2)^2 + 9\beta_1(9\beta_1 - 8\beta_2) > 0
\]

\[
\Delta P = \frac{3(3\beta_1^2 + 2(\beta_1 + 2\beta_2)(a - c))}{4(5(\beta_1 - \beta_2)^2 + 2(a - c)(\beta_1 + \beta_2) + 2\beta_1\beta_2)}
\]

\[
3(3\beta_1^2 + 2(\beta_1 + 2\beta_2)(a - c)) - (4(5(\beta_1 - \beta_2)^2 + 2(a - c)(\beta_1 + \beta_2) + 2\beta_1\beta_2)) = (2\beta_2 - \beta_1)(2(a - c) + 11\beta_1 - 10\beta_2) > 0
\]

\[
\Delta P = \frac{4\beta_1(a - c + \beta_1^2)}{5(\beta_1 - \beta_2)^2 + 2(a - c)(\beta_1 + \beta_2) + 2\beta_1\beta_2}
\]

\[
4\beta_1(a - c + \beta_1^2) - (5(\beta_1 - \beta_2)^2 + 2(a - c)(\beta_1 + \beta_2) + 2\beta_1\beta_2) = -\beta_1^2 + 2(a - c)(\beta_1 - \beta_2) + 8\beta_1\beta_2 - 5\beta_2^2 > 0
\]

Therefore, \(\Delta P^- < \Delta P^+\) and \(\Delta RS^+ < \Delta P^+\).

(4) Investment timing: risk sharing vs full regulation

In order to compare investment timings we do the following computations, considering each time the specific conditions under each relevant parameter threshold, as defined
in Table 2.1, and Assumption 2.4:

- if $F1RS$
  \[
  \frac{\Delta_{RS}}{\Delta P} = \frac{5(\beta_1 - \beta_2)^2 + 2(a - c)(\beta_1 + \beta_2) + 2\beta_1\beta_2}{(2\beta_1 - \beta_2)(2(a - c)(2\beta_1 - \beta_2))} 
  \]
  \[5(\beta_1 - \beta_2)^2 + 2(a - c)(\beta_1 + \beta_2) + 2\beta_1\beta_2 - ((2\beta_1 - \beta_2)(2(a - c)(2\beta_1 - \beta_2))) = 2(2\beta_2 - \beta_1)(a - c) + (2\beta_2 - \beta_1)^2 > 0 \]

- if $F2RS$
  \[
  \frac{\Delta_{RS}}{\Delta P} = \frac{5(\beta_1 - \beta_2)^2 + 2(a - c)(\beta_1 + \beta_2) + 2\beta_1\beta_2}{-72(\beta_1 - \beta_2)^2 + 9(7\beta_1 - 8\beta_2)\beta_2 + 9(7\beta_1 - 9\beta_2)(a - c) - 10(a - c)^2} 
  \]
  \[5(\beta_1 - \beta_2)^2 + 2(a - c)(\beta_1 + \beta_2) + 2\beta_1\beta_2 - \]
  \[(-72(\beta_1 - \beta_2)^2 + 9(7\beta_1 - 8\beta_2)\beta_2 + 9(7\beta_1 - 9\beta_2)(a - c) - 10(a - c)^2) =
  \]
  \[10(a - c)^2 + (83\beta_2 - 61\beta_1)(a - c) + 77(\beta_1 - \beta_2)^2 + \beta_4(72\beta_2 - 61\beta_1) \leq 0 \]

Therefore, for $\beta_1 \leq 2(a - c)/3$, $\Delta_{RS}^* < \Delta_{RS}^*$; for $\beta_1 > 2(a - c)/3$ (the only case in which there is no exclusion with a positive regulated access price), $\Delta_{RS}^* \leq \Delta_{RS}^*$.

(5) Comparative statics

Our comparative statics results, considering the specific conditions under each relevant parameter threshold, as defined in Table 2.1, and Assumption 2.4, are shown below:

- if $P1$
  \[
  \frac{\delta(\Delta P)}{\delta(\beta_2)} = \frac{2(\beta_2 - \beta_1)\gamma}{5\phi} > 0 
  \]

- if $P2$
  \[
  \frac{\delta(\Delta P)}{\delta(\beta_2)} = \frac{(a - c)\gamma}{3\phi} > 0 
  \]

- if $P3$, the OLO is not in the NGN market.

- if $F1$
  \[
  \frac{\delta(\Delta P)}{\delta(\beta_2)} = \frac{-(2(a - c + 2\beta_1 - \beta_2)\gamma}{9\phi} < 0 
  \]

- if $F2$
  \[
  \frac{\delta(\Delta P)}{\delta(\beta_2)} = \frac{(-9(a - c) + 32\beta_1 - 32\beta_2)\gamma}{p\tilde{h}_1} < 0 
  \]

- $RS$
  \[
  \frac{\delta(\Delta_{RS})}{\delta(\beta_2)} = \frac{2(a - c - 4\beta_1 + 5\beta_2)\gamma}{9\phi} 
  \]
  \[2(a - c - 4\beta_1 + 5\beta_2)\gamma > 0 \text{ if } \beta_2 > 4\beta_1/5 - (a - c)/5 \]
  \[2(a - c - 4\beta_1 + 5\beta_2)\gamma \leq 0 \text{ if } \beta_2 \leq 4\beta_1/5 - (a - c)/5 \]

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Therefore, \( \frac{d}{d\beta} \Delta^P > 0; \frac{d}{d\beta} \Delta^F < 0; \) and \( \frac{d}{d\beta} \Delta^{RS} \) changing as shown above.

(6) \textit{Comparison of equilibrium investment timing and socially optimal investment timing}

The comparison of equilibrium investment timing and socially optimal investment timing in the different regulatory regimes give the following results, considering conditions for each parameter range as defined in Table 2.1 and all other assumptions:

- if \( P1 \)
  \[ \frac{\Delta^P}{\Delta^P} = -3((125(a-c)^2) - 30(3\beta_1 + 2\beta_2)(a-c) - 15\beta_1(3\beta_1 + 4\beta_2) - 108\beta_1(\beta_1 - \beta_2)^2) > 0 \]
- if \( P2 \)
  \[ \frac{\Delta^P}{\Delta^P} = -6\beta_1^2 - 4(a-c)(\beta_1 + 2\beta_2) + (a-c)(3\beta_1 + 2\beta_2) + 9\beta_1^2 > 0 \]
- if \( P3 \)
  \[ \frac{\Delta^P}{\Delta^P} = 11(\beta_1 - \beta_2)^2 + 8\beta_1(2\beta_2 - \beta_1) + 8\beta_2(a-c) > 0 \]
- if \( F1 \)
  \[ \frac{\Delta^F}{\Delta^F} = 3(\beta_1^2 + 4\beta_2(a-c) + \beta_2(3\beta_2 - 2\beta_1)) > 0 \]
- if \( F2 \)
  \[ \frac{\Delta^F}{\Delta^F} = 21(a-c)^2 + 180\beta_2(\beta_2 - \beta_1) + 171(\beta_1 - \beta_2)^2 + \beta_2(153\beta_2 - 126\beta_1) > 0 \]
- if \( RS \)
  \[ \frac{\Delta^{RS}}{\Delta^{RS}} = (\beta_1 + 2\beta_2)(4(a-c) + \beta_1 + 2\beta_2) > 0 \]

Therefore, \( \Delta^P < \Delta^{PW*}; \Delta^F < \Delta^{FW*}; \) and \( \Delta^{RS} < \Delta^{RSW*}. \)

2.A.4 \textbf{Proof of Proposition 2.4}

Expected consumer welfare is defined as:

\[ E(CS^i) = \Delta^i(CS^C) + (1 - \Delta^i)E(CS^L) \]
\[ = \Delta^i \left( \frac{(Q^C)^2}{2} \right) + (1 - \Delta^i) \left( \gamma \left( \frac{(Q^{WS})^2}{2} \right) + (1 - \gamma) \left( \frac{(Q^{WS})^2}{2} \right) \right) \]

with \( Q^i = q^l_1 + q^l_2. \)
expected total welfare is defined as:

\[
E(W^t) = \Delta^t \left( \frac{(Q^e)^2}{2} + \frac{q^e}{2} - \Delta^t \phi / 2 + (q^e)^2 \right) +
\]

\[
(1 - \Delta^t) \left( \frac{(Q^{f,s})^2}{2} + (q^{f,s})^2 + \Delta^t \phi / 2 + (q^{f,s})^2 \right) +
\]

\[
(1 - \gamma) \left( \frac{(Q^{l,s})^2}{2} + (q^{l,s})^2 - \Delta^t \phi / 2 + (q^{l,s})^2 \right)
\]

Our analysis reveal the following ranking of expected total welfare and expected consumer welfare, respectively. Notice that the results are broken down according to the relevant parameter thresholds defined in Table 2.1.

\[
\begin{cases}
E(W^{RS}) > E(W^F) > E(W^P) & \text{if } \beta_2, \beta_1 \text{ s.t. } P1/P2F1RS, \text{ with } \beta_2 \geq \beta_1 \\
E(W^{RS}) \leq E(W^P) > E(W^F) & \text{if } \beta_2, \beta_1 \text{ s.t. } P2F1RS, \text{ with } \beta_2 < \beta_1 \\
E(W^{RS}) > E(W^F); E(W^{RS}) \leq E(W^P); E(W^F) \leq E(W^P) & \text{if } \beta_2, \beta_1 \text{ s.t. } P3F1RS \\
E(W^P) > E(W^{RS}) \leq E(W^P) & \text{if } \beta_2, \beta_1 \text{ s.t. } P3F2RS
\end{cases}
\]

We now proceed by analysing each single statement contained in Proposition 2.4.

1. **Consumer welfare: risk sharing vs partial regulation**
   In order to compare consumer welfare outcomes it is sufficient to compare total quantities. So we check under each of the specific parameter thresholds, defined in Table 2.1 and find:

   \[
   \begin{cases}
   \frac{Q^P}{Q_{R}^P} < 0 & \text{if } P1RS \\
   \frac{Q^P}{Q_{R}^P} \leq 0 & \text{if } P2RS \\
   \frac{Q^P}{Q_{R}^P} < 0 & \text{if } P3RS
   \end{cases}
   \]

   Therefore, \( E(C^{RS}) > E(C^F) \).

2. **Total welfare: risk sharing vs partial regulation and (3) Total welfare and consumer welfare: partial regulation vs full regulation**
   From the results above, we derive that, in all cases in which \( \beta_2 \geq \beta_1 \), namely \( P1F1RS \) and \( P2F1RS \) (only for the part in which \( \beta_2 \geq \beta_1 \)); \( W^{RS} > W^F \) \( W^F \). Furthermore, when the incumbent is better than the OLO by a great extent and the regulated access price is positive, case \( P3F2 \), we have: \( W^F > W^F \).

3. **Total welfare and consumer welfare: risk sharing vs full regulation**
   In order to compare consumer welfare outcomes it is sufficient to compare total quantities. So we check under each of the specific parameter thresholds, defined in Table 2.1 and find:

   \[
   \begin{cases}
   \frac{Q^P}{Q_{R}^P} < 0 & \text{if } P1RS \\
   \frac{Q^P}{Q_{R}^P} \leq 0 & \text{if } P2RS (happening without exclusion only if } \beta_1 > 2(a - c)/3
   \end{cases}
   \]

\[17\] Since expressions are cumbersome, detailed equations are available from the authors upon request.
Therefore, \( E(CS^{RS}) > E(CS^F) \) when the access price is regulated at cost, and the relationship is ambiguous when the access price is positive. Also, from the results above we obtain that only in case \( F1: W^{RS} > W^F \).

(5) Total welfare and consumer welfare: full regulation ranking From the results above, we can conclude that, in all cases in which there is no exclusion of the OLO from the NGN market, namely \( P1F1RS \) and \( P2F1RS: W^{RS} > W^F \) and \( W^P > W^F \); \( CS^{RS} > CS^F \) and \( CS^P > CS^F \).

**Appendix 2.B Extension: Full Regulation with Single Access Charge**

In this Extension we look at the case in which the regulator chooses a single access charge to the NGN independent of demand.

**Stage 2: Retail market competition**

Expected demand functions are unchanged with respect to our basic model:

\[
E[p^F_i] = \gamma \left( p^F_i - c_i - q^F_i - \hat{r}^F q^F_i \right) + (1 - \gamma) \left( p^F_i - c_i - q^F_i - \hat{r}^F q^F_i \right)
\]

Expected profit functions faced by the firms in the NGN market are now:

\[
E[\pi^F_i] = \gamma \left( (p^F_i - c_i - q^F_i - \hat{r}^F q^F_i) \right) + (1 - \gamma) \left( (p^F_i - c_i - q^F_i - \hat{r}^F q^F_i) \right) - (\Delta^F)\phi/2
\]

where \( \hat{r}^F \) is the single access charge set by the regulator for the NGN; \( p^F_i \) is the demand in case of success, with \( i = 1, 2 \), and \( h = s, f \) for success and failure; \( q^F_i \) is the quantity, with \( i = 1, 2 \), and \( h = s, f \) for success and failure; \( \Delta^F \) is the investment timing chosen by the incumbent in the full regulation case; \( \phi \) is a positive cost parameter.

The copper network market looks exactly the same as in our basic model.

The first-order conditions give the following equilibrium quantities in the retail market for the NGN:

- in case of successful investment
  \[
  \begin{align*}
  q^F_{s1}^* &= \frac{a - c_1 + 2\Delta s - \hat{r}^F + 2F}{3} \\
  q^F_{s2}^* &= \frac{a - c_2 + 2\Delta s - \hat{r}^F - 2F}{3}
  \end{align*}
  \]

- in case of unsuccessful investment
  \[
  \begin{align*}
  q^F_{f1}^* &= \frac{a - c_1 + \hat{r}^F}{3} \\
  q^F_{f2}^* &= \frac{a - c_2 + 2\hat{r}^F}{3}
  \end{align*}
  \]

We maintain the fundamental assumptions of the model on the parameters, namely:

**Assumption 2.3.** \( r^F \geq 0 \) and \( r^C = 0 \)

**Assumption 2.4.** \( 2\beta_i \geq \beta_j, \forall i, j = 1, 2 \) with \( i \neq j \)
Assumption 2.4 is sufficient to ensure that both firms will be active in the market, provided that the access charge is not too high.

The OLO participation condition to the NGN market, considered the new equilibrium quantities, is the following:

\[
r^F \leq \frac{a - c + \gamma(2\beta_2 - \beta_1) - \sqrt{(a - c)^2 - \gamma(1 - \gamma)(2\beta_2 - \beta_1)^2}}{2}
\]

Notice that for \( \gamma = 1 \), this condition equals Condition 2.5 in our basic model.

**Stage 1: the regulator sets the access price to the NGN**

The regulator sets a single access fee to the NGN in order to maximise social welfare. Its objective function after investment is the following:

\[
E(W^{NF}) = \gamma \left( \frac{(q_1^{F*} + q_2^{F*})^2}{2} + (q_1^{F*})^2 + r^{F} q_2^{F*} - \Delta \phi/2 + (q_2^{F*})^2 \right) (1 - \gamma) \left( \frac{(q_1^{F*} + q_2^{F*})^2}{2} + (q_1^{F*})^2 + r^{F} q_2^{F*} - \Delta \phi/2 + (q_2^{F*})^2 \right)
\]

The first-order condition with respect to \( r^F \) gives the access price as:

\[
r^F = c - a + \gamma(4\beta_1 - 5\beta_2)
\]

\( c - a < 0 \) is a necessary condition for a broadband market to exist. If \( \beta_1 > \beta_2 \) so much that \( \gamma(4\beta_1 - 5\beta_2) > a - c \), then the solution to the first-order condition given by the expression above is positive, \( r^{F*} > 0 \), and it constitutes the regulated access price.

Notice that the single access charge makes it less likely that the socially optimal \( r^{F*} \) is positive, i.e. the range of parameters shrinks with respect to our basic model, meaning that the chance for the incumbent to be awarded of its higher ability in offering service, and thereby to actually benefit of its own investment in case of success, decreases.

If, otherwise, the incumbent is worse than the OLO in offering value-added services, \( \beta_1 \leq \beta_2 \), or if it is better in offering value-added services but not by a great extent, \( \beta_1 > \beta_2 \) but \( \gamma(4\beta_1 - 5\beta_2) < a - c \), the solution to the first-order condition is lower than the incumbent’s marginal cost of network operations, i.e. \( r^{F*} < 0 \). This contradicts our Assumption 3.1, according to which \( r^{F*} \geq 0 \), so in this case we will impose \( r^{F*} = 0 \), such that the optimal regulated access price will be set equal to the marginal cost.

By substituting the values for \( r^{F*} \) into the expressions for the equilibrium quantities, we obtain the following expected quantities:

- for \( \gamma(4\beta_1 - 5\beta_2) \leq a - c \) (i.e. when \( r^{F*} = 0 \))

\[
E(q_1^{F*}) = \gamma \left( \frac{a - c + 2\beta_1 - \beta_2}{3} \right) + (1 - \gamma) \left( \frac{a - c}{3} \right)
\]

\[
E(q_2^{F*}) = \gamma \left( \frac{a - c + 2\beta_2 - \beta_1}{3} \right) + (1 - \gamma) \left( \frac{a - c}{3} \right)
\]

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• otherwise (i.e. when \( \hat{r}^{F*} \) is above cost)

\[
E(q_1^{F*}) = \gamma \left( \frac{2\beta_1 - \beta_2 + \gamma(4\beta_1 - 5\beta_2)}{3} \right) + \\
(1 - \gamma) \left( \frac{\gamma(4\beta_1 - 5\beta_2)}{3} \right)
\]

\[
E(q_2^{F*}) = \gamma \left( a - c + \frac{2\beta_2 - \beta_1 - 2\gamma(4\beta_1 - 5\beta_2)}{3} \right) + \\
(1 - \gamma) \left( a - c - \frac{2\gamma(4\beta_1 - 5\beta_2)}{3} \right)
\]

Comparing the profits obtainable by the OLO with the outside option with those obtainable by participating to the NGN, in case of failure (the stricter condition), we find that the OLO decides to opt for the NGN in case of failure as long as the following condition is satisfied:

\[
\left( \frac{a - c}{3} \right)^2 \leq \left( a - c - \frac{2\gamma(4\beta_1 - 5\beta_2)}{3} \right)^2
\]

where the left hand side represents the OLO’s profits in the outside option and the right hand side represents the OLO’s profits in the NGN market, in case of failure. From the above condition we draw that whenever \( \hat{r}^{F*} \) is above cost, in case of failure, the OLO prefers to stay out of the NGN market.

Furthermore, it is clear that the condition for the incumbent to gain revenues from leasing the network in the upstream segment identifies a smaller range of parameter values compared to our basic model.

Figure 2.5: Full Regulation: Case of failure

Graphically, the situation would be twofold. In case of failure, we would have a situation like in Figure 2.5, while in case of success we would have a situation like in Figure 2.6. The parameter threshold for exclusion in case of success comes from the OLO’s participation condition after simple algebra.

*Stage 0: the incumbent chooses the investment timing*
The incumbent will have different objective functions depending on the parameters. In particular, when \( \gamma(4\beta_1 - 5\beta_2) \leq a - c \) we have that \( \hat{r}^F = 0 \). Therefore the incumbent makes no profit in the upstream market and the situation is unchanged with respect to our basic model.

When \( \gamma(4\beta_1 - 5\beta_2) > a - c \), we have that \( \hat{r}^F > 0 \), then the incumbent’s objective function is:

\[
\max_{\Delta F} E(\pi^F) = (1 - \hat{\Delta}^F) \left( \frac{a - c}{3} \right)^2 + \\
\hat{\Delta}^F \left( \gamma((2\beta_1 - \beta_2 + \gamma(4\beta_1 - 5\beta_2))^2 + \\
(c - a + \gamma(4\beta_1 - 5\beta_2))(a - c + \gamma(4\beta_1 - 5\beta_2)) + \\
(1 - \gamma) \left( \frac{\gamma(4\beta_1 - 5\beta_2)^2}{3} \right)^2 \right) - (\hat{\Delta}^F)^2 \phi/2
\]

The resulting investment timing, \( \hat{\Delta}^F \), is:

\[
\hat{\Delta}^F = \begin{cases} 
\frac{(2(a - c)(2\beta_1 - \beta_2 + (2\beta_1 - \beta_2)^2)\gamma}{9\phi} & \text{if } \gamma(4\beta_1 - 5\beta_2) \leq a - c \\
\frac{(29\beta_2 - 19\beta_1)(4\beta_1 - 5\beta_2)^2 + 9\gamma(7\beta_1 - 9\beta_2)(a - c) + \gamma(2\beta_1 - \beta_2)^2 - 10(a - c)^2\gamma}{9\phi} & \text{otherwise}
\end{cases}
\]

This is the incumbent optimal investment timing as long as the conditions \(((2(a - c)(2\beta_1 - \beta_2 + (2\beta_1 - \beta_2)^2)\gamma)/(9\phi) \leq 1 \) and \((29\beta_2 - 19\beta_1)(4\beta_1 - 5\beta_2)^2 + 9\gamma(7\beta_1 - 9\beta_2)(a - c) + \gamma(2\beta_1 - \beta_2)^2 - 10(a - c)^2\gamma)/(9\phi) \leq 1 \) are satisfied.

Notice that if \( \gamma = 1 \) the optimal timing here coincides with the optimal timing in our basic model, therefore, being \( \gamma \) less or equal to 1, we observe that, in case of single regulated access charge, the investment timing is further delayed with respect to the case with access charge dependent on demand, as analysed in our basic model.
Bibliography


